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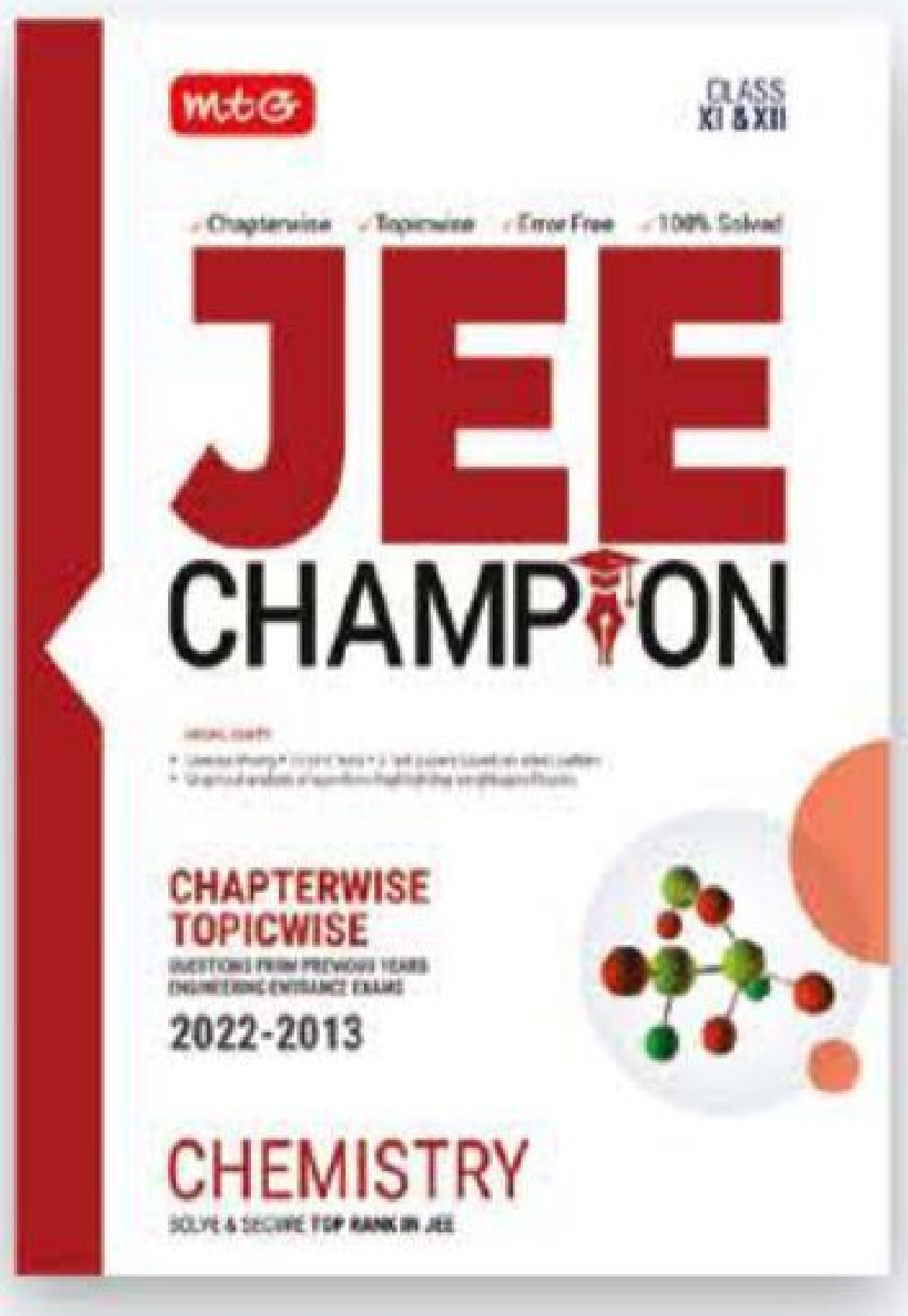
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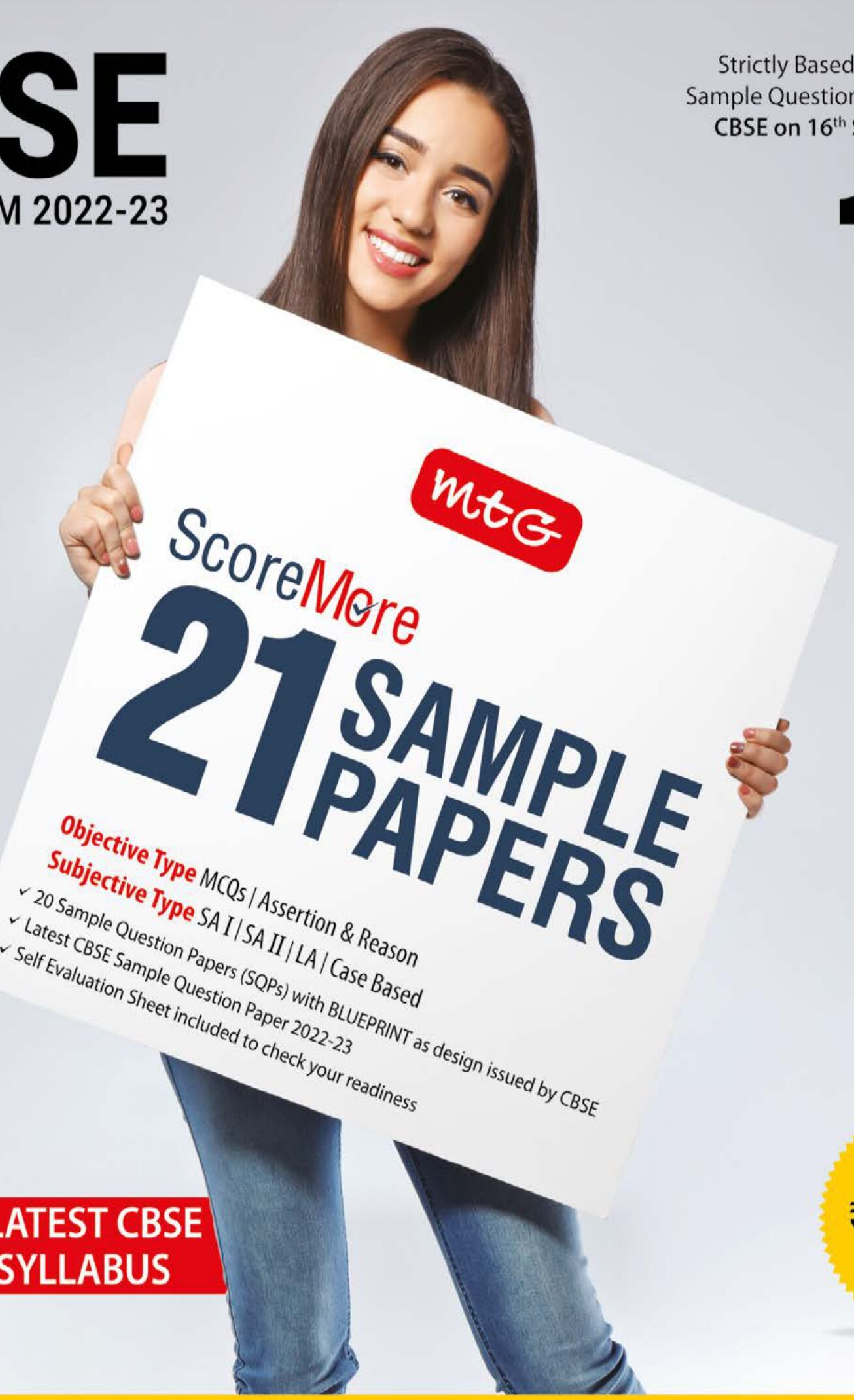
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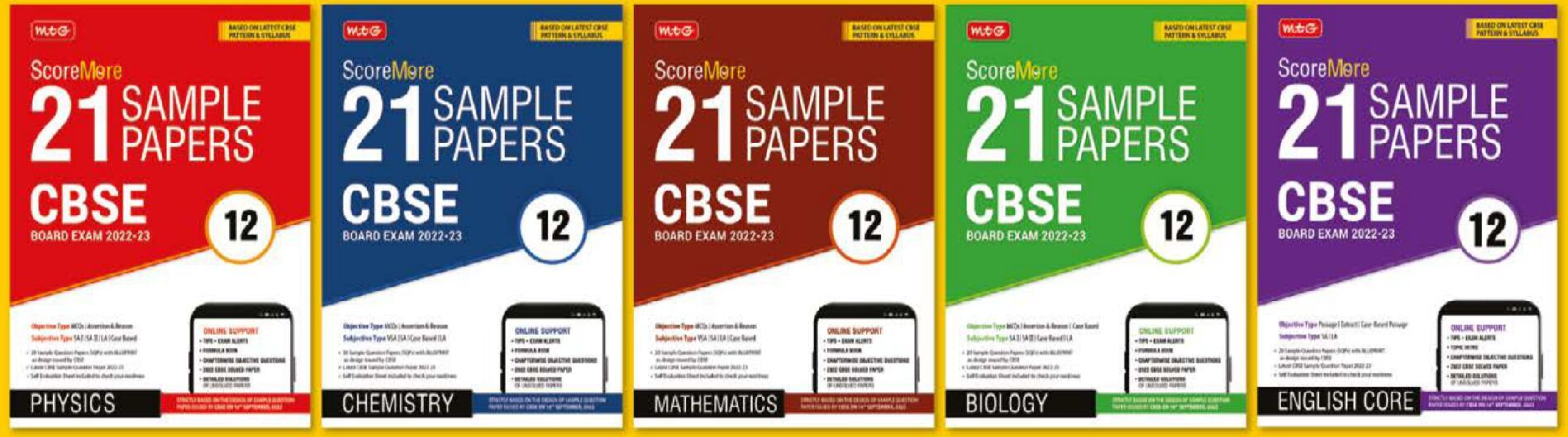

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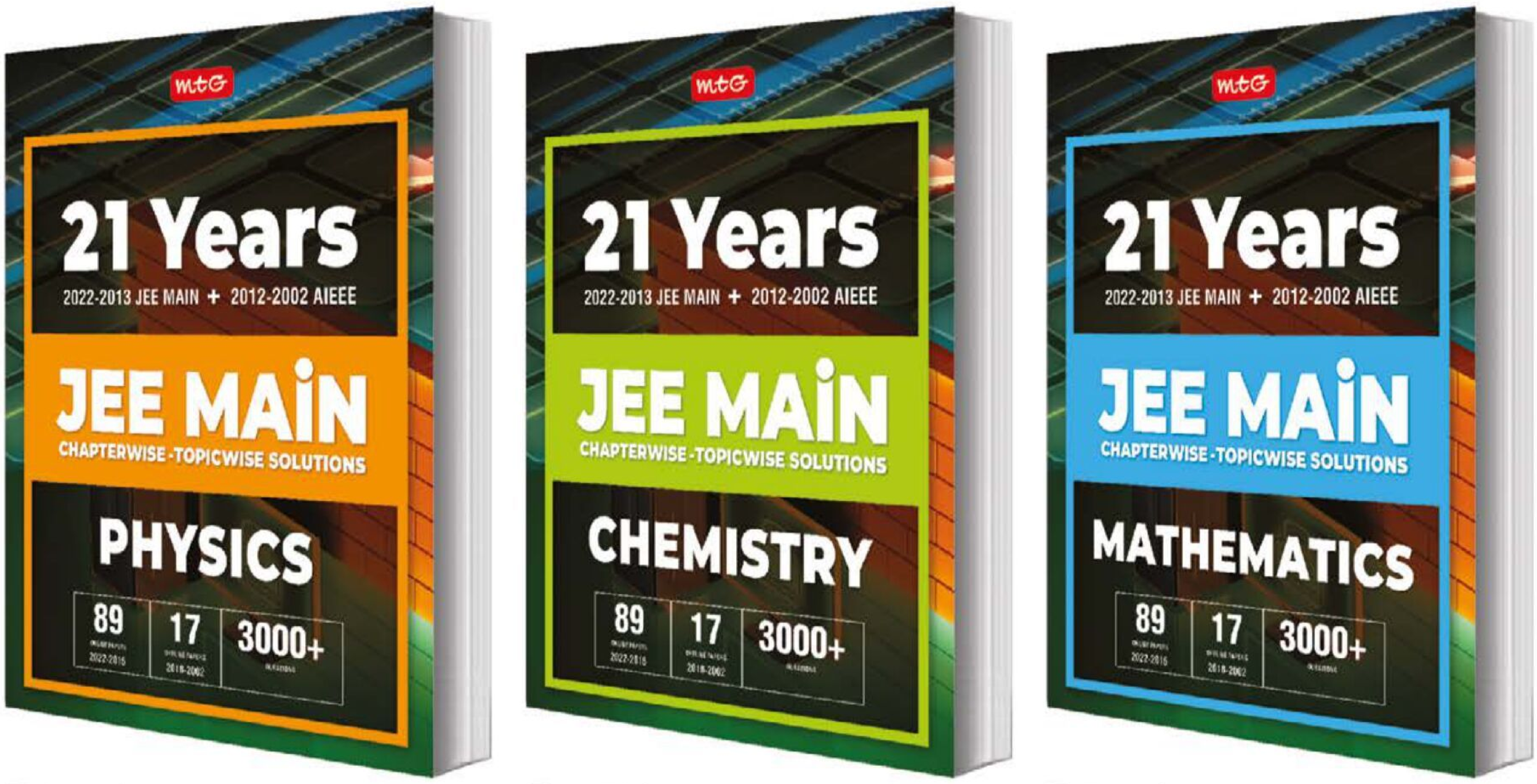
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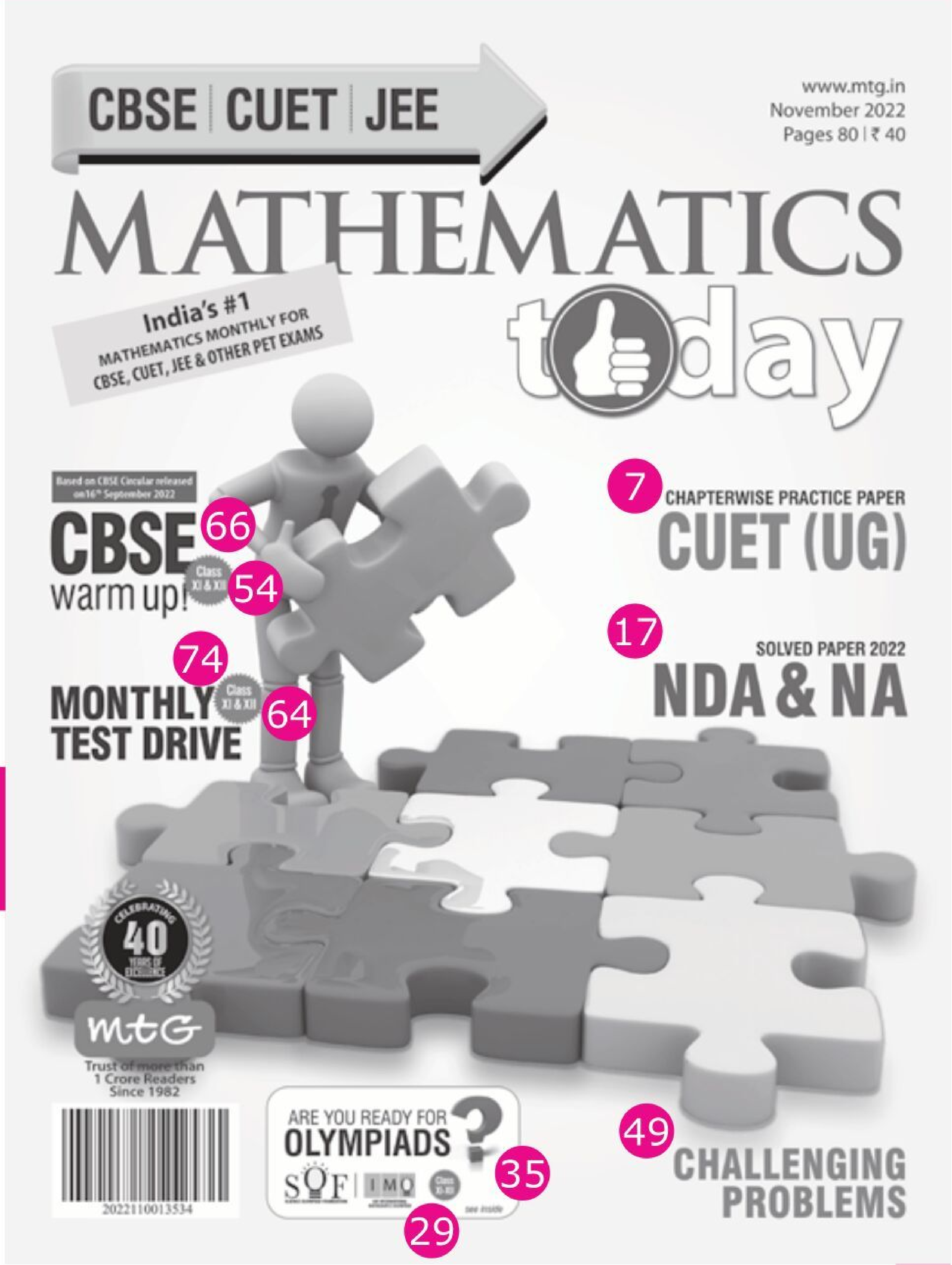

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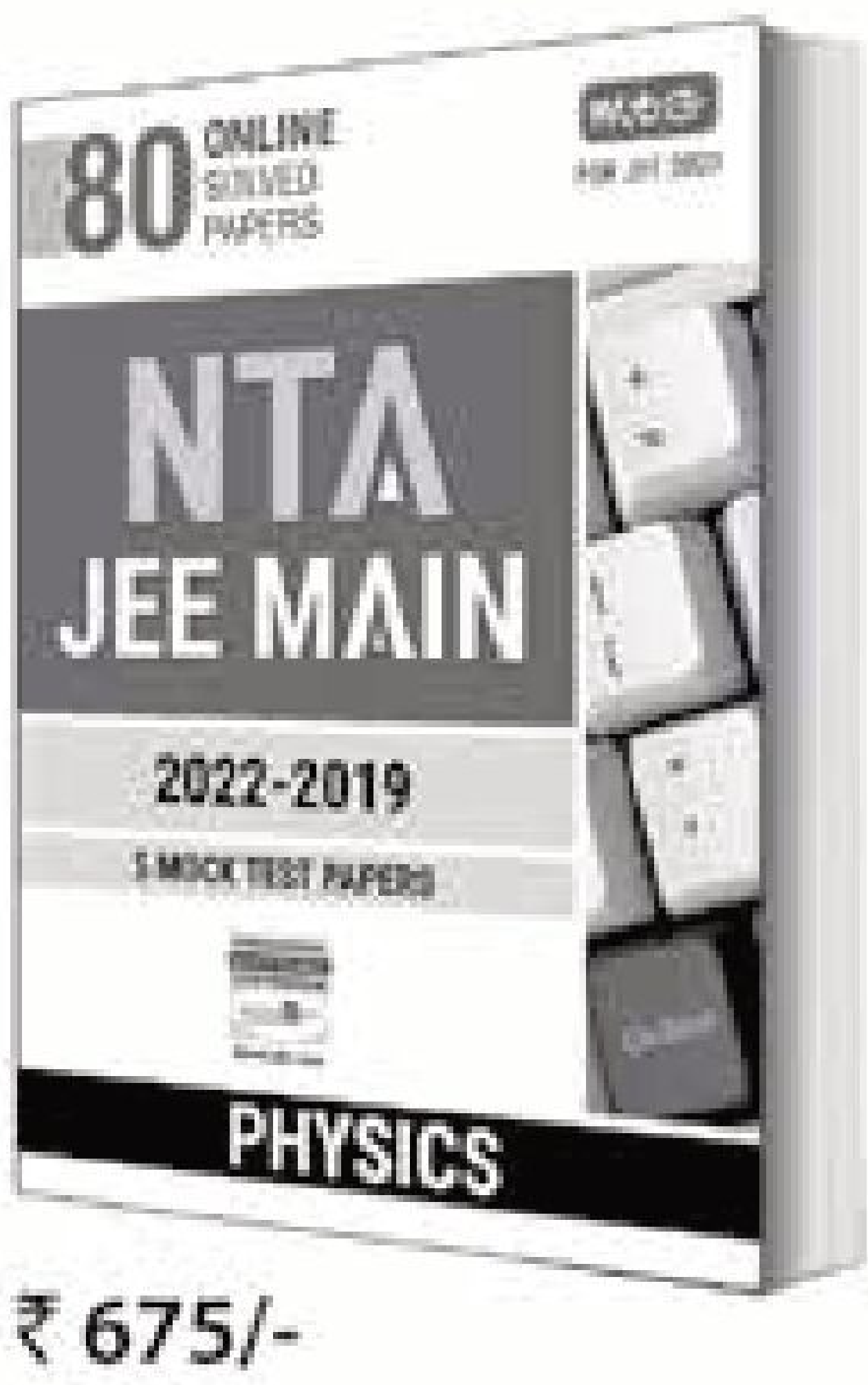
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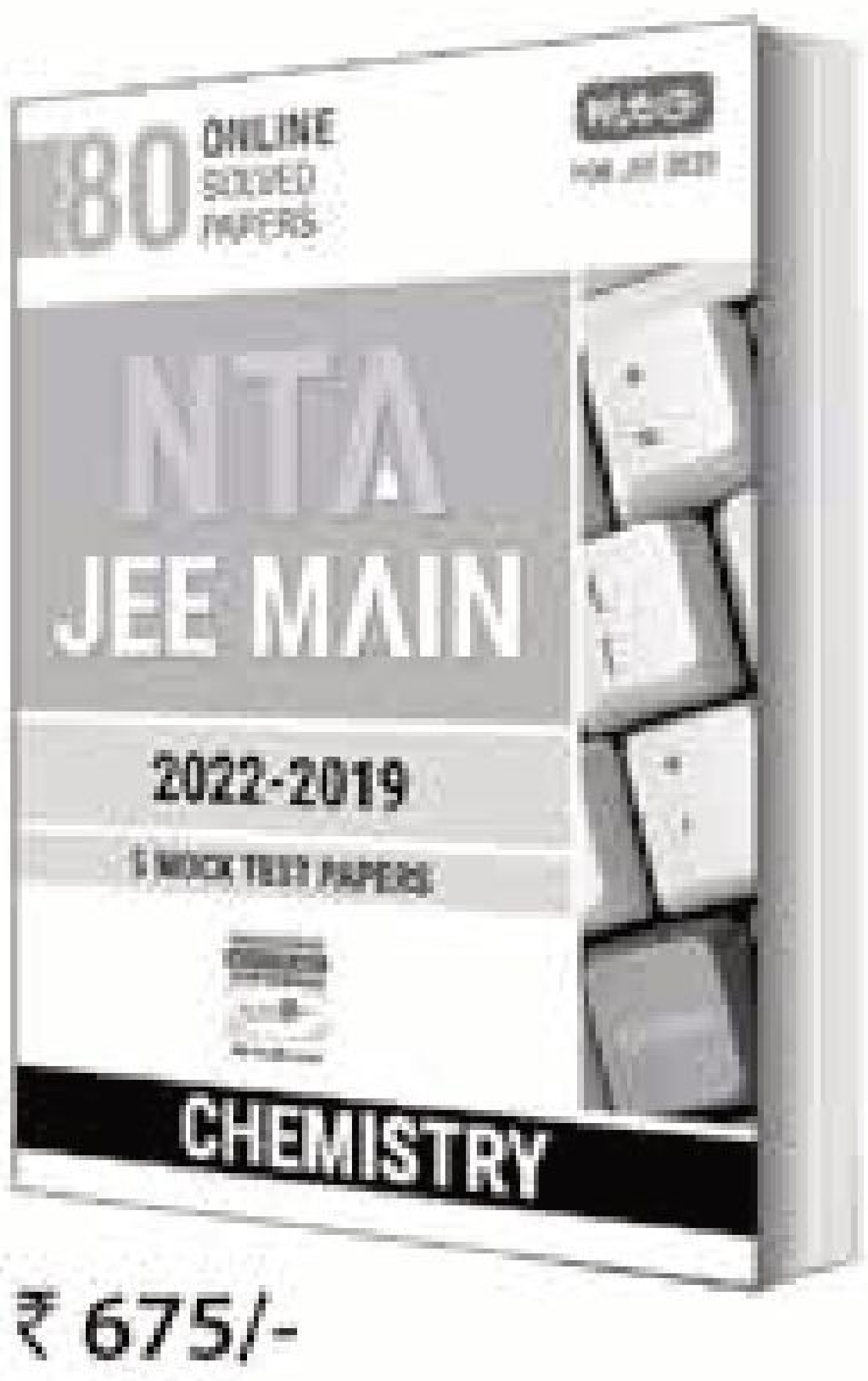
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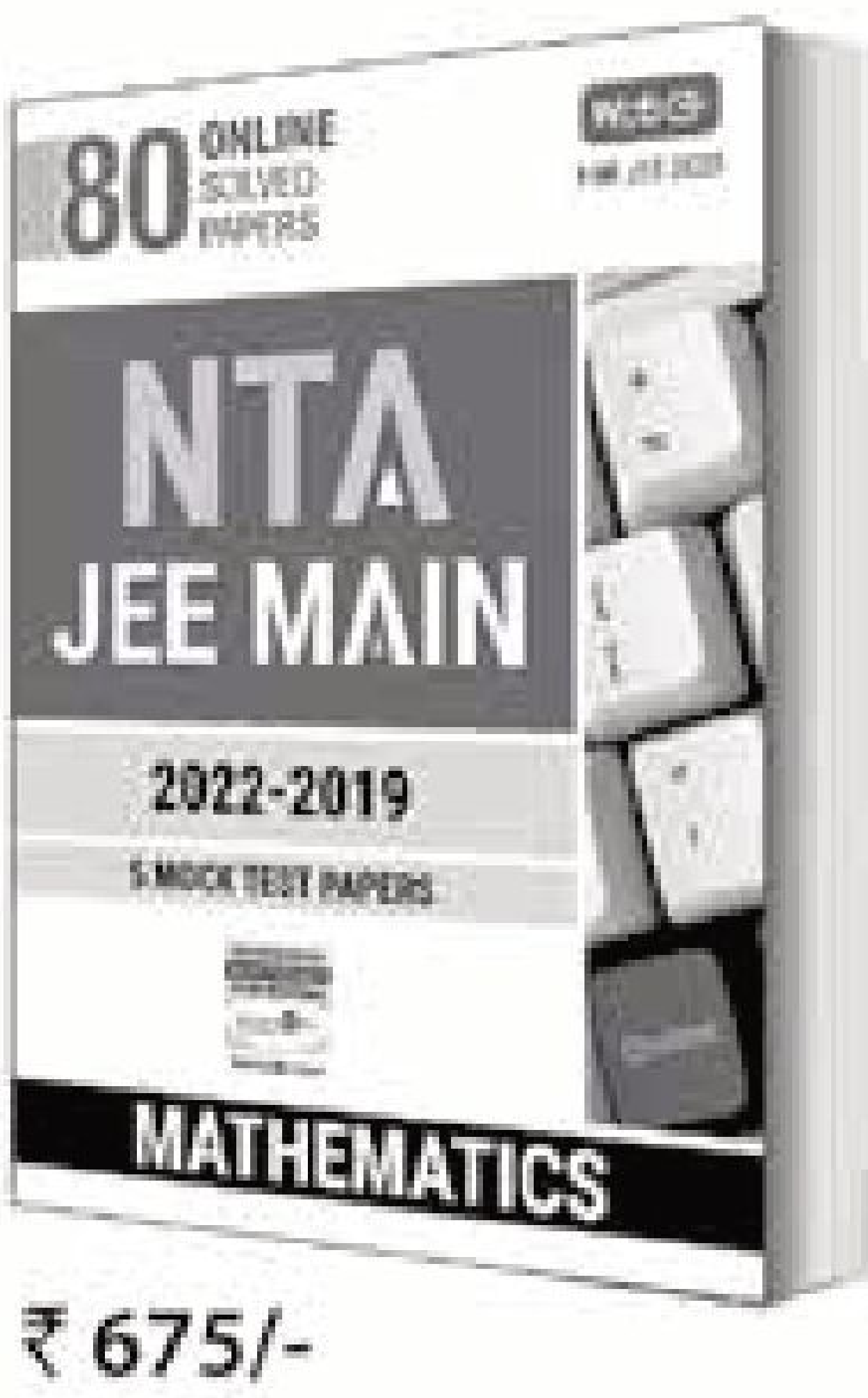
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# CHAPTERWISE PRACTICE PAPER 2023

# CUET (UG)

## SERIES - III Continuity and Differentiability | Application of Derivatives

***Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.***

**Time Allowed :** 45 Minutes

**Maximum Marks :** 200

### Multiple Choice Questions (MCQs)

**1.** Let $f(x)=\begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}, & x<0 \\ c, & x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & x>0 \end{cases}$

If $f(x)$ is continuous at $x = 0$, then

(a) $a + c = 0, b = 1$ (b) $a + c = 1, b \in R$

(c) $a + c = -1, b \in R$ (d) $a - c = -1, b = -1$

**2.** If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{x+1}{x}$ (b) $\dfrac{1}{1+x}$ (c) $\dfrac{-1}{(1+x)^2}$ (d) $\dfrac{x}{1+x}$

**3.** If $f(x) = \dfrac{\sqrt{4+x}-2}{x}$, $x \neq 0$ be continuous at $x = 0$, then $f(0) =$

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$ (c) 2 (d) $\dfrac{3}{2}$

**4.** If $x \sin(a + y) = \sin y$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{\sin^2(a+y)}{\sin a}$ (b) $\dfrac{\sin a}{\sin^2(a+y)}$

(c) $\dfrac{\sin(a+y)}{\sin a}$ (d) $\dfrac{\sin a}{\sin(a+y)}$

**5.** The function $f(x)=\begin{cases} |x-3|, & x \geq 1 \\ \dfrac{x^2}{4}-\dfrac{3x}{2}+\dfrac{13}{4}, & x<1 \end{cases}$ is

(a) continuous at $x = 1$ (b) differentiable at $x = 1$

(c) continuous at $x = 3$ (d) All of these

**6.** If $\sqrt{x+y}+\sqrt{y-x}=a$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{\sqrt{x+y}-\sqrt{y-x}}{\sqrt{y-x}+\sqrt{x+y}}$ (b) $\dfrac{2\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$

(c) $\dfrac{x+y+\sqrt{xy}}{\sqrt{x+y}}$ (d) $\dfrac{x^2+y^2+2xy}{x^2+y^2}$

**7.** The position of a point in time '$t$' is given by $x = a + bt - ct^2$, $y = at + bt^2$. Its acceleration at time '$t$' is

(a) $b - c$ (b) $b + c$

(c) $2b - 2c$ (d) $2\sqrt{b^2+c^2}$

**8.** The slope of the tangent to the curve $y^2e^{xy} = 9e^{-3}x^2$ at $(-1, 3)$ is

(a) $\dfrac{-15}{2}$ (b) $\dfrac{-9}{2}$ (c) 15 (d) $\dfrac{15}{2}$

**9.** If $x^my^n = (x + y)^{m+n}$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{x+y}{xy}$ (b) $xy$ (c) $\dfrac{x}{y}$ (d) $\dfrac{y}{x}$

**10.** The equation of the normal to the curve $y = \sin x$ at $(0, 0)$ is

(a) $x = 0$ (b) $x + y = 0$

(c) $y = 0$ (d) $x - y = 0$

**11.** If $f(x) = (1 + x)^n$, then the value of

$f(0)+f'(0)+\dfrac{f''(0)}{2!}+\ldots+\dfrac{f^{(n)}(0)}{n!}$ is equal to

(a) $2^{n-1}$ (b) $2n$ (c) $n$ (d) $2^n$

**12.** If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of $\alpha$ for which Rolle's theorem can be applied in $[0, 1]$, is

(a) $-1$ (b) $1/2$ (c) $-1/2$ (d) 0

**13.** $\sin^p\theta\cos^q\theta$ attains a maximum, when $\theta =$

(a) $\tan^{-1}\sqrt{\frac{p}{q}}$ (b) $\tan^{-1}\left(\frac{p}{q}\right)$

(c) $\tan^{-1}q$ (d) $\tan^{-1}\left(\frac{q}{p}\right)$

**14.** The function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$ is strictly increasing for all $x$, if

(a) $k > 3/2$ (b) $k \geq \frac{3}{2}$ (c) $k < \frac{3}{2}$ (d) $k \leq \frac{3}{2}$

**15.** Let $f(x)$ satisfy the requirements of Mean Value theorem in $[0, 2]$. If $f(0) = 0$ and $f'(x) \leq \frac{1}{2}$ for all $x$ in $[0, 2]$, then

(a) $|f(x)| \leq 2$ (b) $f(x) \leq 1$

(c) $f(x) = 2x$

(d) $f(x) = 3$ for atleast one $x$ in $[0, 2]$

**16.** If there is an error of $a$%, where $a$ is very small positive quantity in measuring the edge of a cube, then percentage error in its surface area is

(a) $2a$% (b) $\frac{a}{2}$%

(c) $3a$% (d) None of these

**17.** If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y}{x}$ (b) $\frac{y}{x}$ (c) $-\frac{x}{y}$ (d) $\frac{x}{y}$

**18.** If the distance $S$ covered by a particle in time $t$ is proportional to the cube root of its velocity, then the acceleration is

(a) a constant (b) $\propto S^3$

(c) $\propto \frac{1}{S^3}$ (d) $\propto S^5$

**19.** If $\sin y + e^{-x\cos y} = e$, then $\frac{dy}{dx}$ at $(1, \pi)$ is equal to

(a) $\sin y$ (b) $-x\cos y$

(c) $e$ (d) $\sin y - x\cos y$

**20.** The equation of the tangent to the curve $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ at the point $(x_1, y_1)$ is $\frac{x}{\sqrt{ax_1}} + \frac{y}{\sqrt{by_1}} = k.$. Then the value of $k$ is

(a) 2 (b) 1 (c) 3 (d) 7

**21.** Let $k$ and $K$ be the minimum and the maximum values of the function $f(x) = \frac{(1+x)^{0.6}}{1+x^{0.6}}$ in $[0, 1]$ respectively, then the ordered pair $(k, K)$ is equal to

(a) $(1, 2^{0.6})$ (b) $(2^{-0.4}, 2^{0.6})$

(c) $(2^{-0.6}, 1)$ (d) $(2^{-0.4}, 1)$

**22.** The function $f(x) = 1 - x^3 - x^5$ is decreasing for

(a) $1 \leq x \leq 5$ (b) $x \leq 1$

(c) $x \geq 1$ (d) all values of $x$

**23.** If the volume of a sphere is increasing at a constant rate, then the rate at which its radius is increasing, is

(a) a constant

(b) proportional to the radius

(c) inversely proportional to the radius

(d) inversely proportional to the surface area

**24.** If the tangent at $P(1, 1)$ on $y^2 = x(2 - x)^2$ meets the curve again at $Q$, then $Q$ is

(a) $(2, 2)$ (b) $(-1, -2)$

(c) $\left(\frac{9}{4}, \frac{3}{8}\right)$ (d) None of these

**25.** If $y^2 = 100\tan^{-1}x + 45\sec^{-1}x + 100\cot^{-1}x + 45\operatorname{cosec}^{-1}x$, then $\frac{dy}{dx} =$

(a) $\frac{x^2-1}{x^2+1}$ (b) $\frac{x^2+1}{x^2-1}$ (c) 1 (d) 0

**26.** If $y = \tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) 2 (b) 1 (c) 0 (d) $-1$

**27.** If $a + b + c = 0$, then the quadratic equation $3ax^2 + 2bx + c = 0$ has

(a) at least one root in $(0, 1)$

(b) one root in $(2, 3)$ and other in $(-2, -1)$

(c) imaginary roots

(d) none of these

**28.** The function $f(x) = x + \frac{4}{x}$ has

(a) a local maxima at $x = 2$ and local minima at $x = -2$

(b) local minima at $x = 2$, and local maxima at $x = -2$

(c) absolute maxima at $x = 2$ and absolute minima at $x = -2$

(d) absolute minima at $x = 2$ and absolute maxima at $x = -2$

**29.** If two positive numbers $x$ and $y$ are such that $x + y = 60$ and $xy^3$ is maximum, then the numbers $x$ and $y$ are respectively

(a) 13, 47 (b) 28, 32 (c) 12, 48 (d) 15, 45

**30.** $\frac{d}{dx}\left\{\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)\right\}$ is equal to

(a) $-\frac{2}{1+x^2}, x \neq 0$ (b) $\frac{2(1+x)}{1+x^2}, x \neq 0$

(c) $\frac{2(1-x^2)}{(1+x^2)|1-x^2|}, x \neq \pm 1, 0$

(d) None of these

**31.** If $f(x) = \sin x - \cos x$, the interval in which function is decreasing in $0 \leq x \leq 2\pi$, is

(a) $\left[\frac{5\pi}{6}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(c) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ (d) None of these

**32.** Let $f(x)$ and $g(x)$ be differentiable functions for $0 \leq x \leq 1$, such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$. Let there exist a real number $c$ in $(0, 1)$ (where LMVT is applicable) such that $f'(c) = 2g'(c)$, then the value of $g(1)$ must be

(a) 1 (b) 2 (c) –2 (d) –1

**33.** Tangents to the curve $y = x^3 + 3x$ at $x = -1$ and $x = 1$ are

(a) parallel
(b) intersecting obliquely but not at an angle of 45°
(c) intersecting at right angles
(d) intersecting at an angle of 45°

**34.** The point on the curve $y^2 = x$, where tangent makes an angle of 45° with the $X$-axis is

(a) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (c) (4, 2) (d) (2, – 2)

## Assertion & Reasoning Based MCQs

**For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.**

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.
(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
(c) If Statement-I is true but Statement-II is false.
(d) If Statement-I is false and Statement-II is true.

**35.** **Statement-I :** The absolute maximum and minimum values of $f(x) = x^2\sqrt{1+x}$ in $\left[-1, \frac{1}{2}\right]$ are $\frac{\sqrt{6}}{8}$ and 0 respectively.

**Statement-II :** Let $f$ be a differentiable function on $I$ and $x_0$ be any interior point of $I$. If $f$ attains its absolute maximum or minimum value at $x_0$, then $f'(x_0) = 0$.

**36.** Consider the function $f(x) = x^{1/3}, x \in R$.

**Statement-I :** $f''(0) = 0$.

**Statement-II :** $f$ has a point of inflexion at $x = 0$.

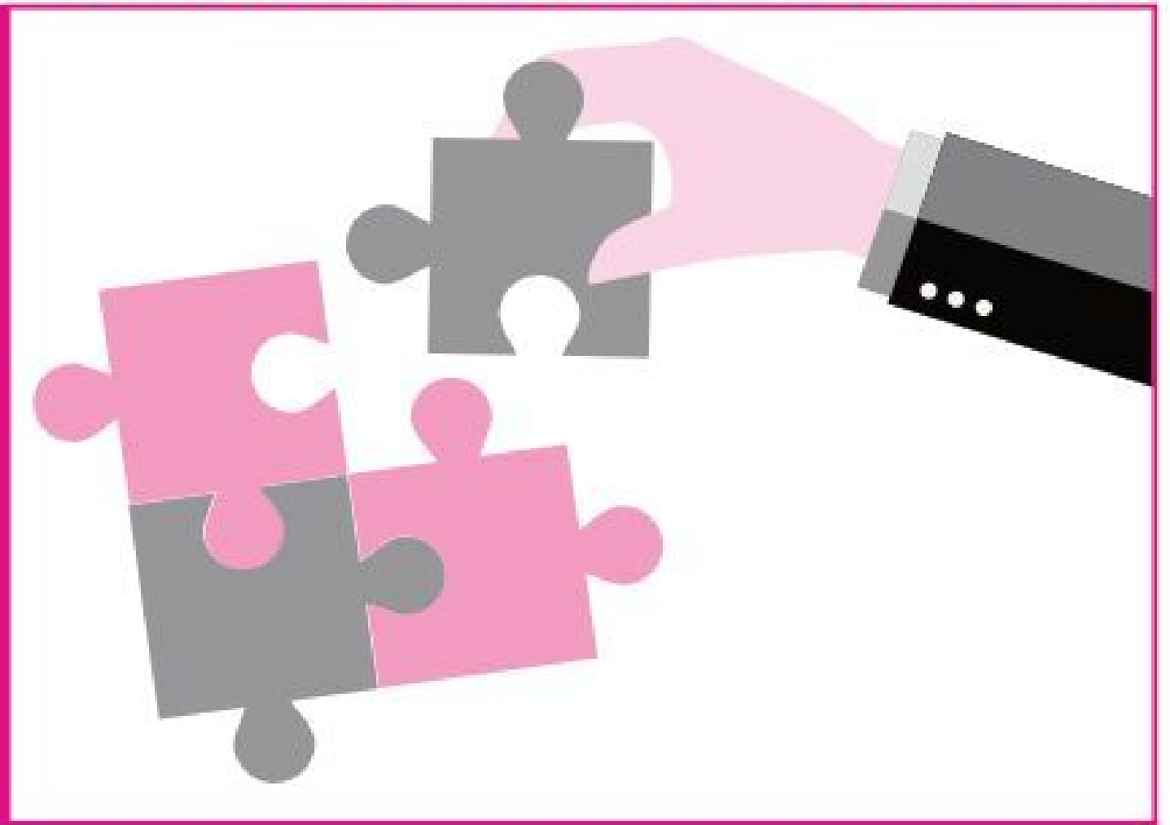
**37.** **Statement-I :** If $y = \cot^{-1}\left(\frac{1+x\sqrt{x}}{\sqrt{x}-x}\right)$, then $\frac{dy}{dx} = \frac{-1}{1+x^2} + \frac{1}{2\sqrt{x}(1+x)}$

**Statement-II :** $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ and $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$.

**38.** **Statement-I :** If $x = at^2$ and $y = 2at$, then $\left[\frac{d^2y}{dx^2}\right]_{t=2} = \frac{-1}{16a}$.

**Statement-II :** $\frac{d^2y}{dx^2} = \left(\frac{dy}{dt}\right)^2 \times \left(\frac{dt}{dx}\right)^2$

# PUZZLE CORNER



## MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

| | | | | | |
|---|---|---|---|---|---|
| 1– | | 108÷ | 5+ | | 50× |
| 15+ | 1– | | | | |
| | | 1– | | 3– | |
| | | 2– | | 3÷ | 3– |
| 2– | 4– | | 2÷ | | |
| | 4– | | | 3÷ | |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

## Matrix-Match

**(39-40) : Match Column I with Column II and select the correct answer by choosing an appropriate option.**

**39.**

| | Column I | | Column II |
|---|---|---|---|
| (P) | If $x \neq 0$ and $y = \log_e\lvert 2x\rvert$, then $\frac{dy}{dx}$ is equal to | (1) | $\frac{\log_{10} e}{2x^2}$ |
| (Q) | Derivative of $\log_{10} x$ with respect to $x^2$ is | (2) | $\frac{y}{x}$ |
| (R) | If $x^y y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is | (3) | $\frac{1}{x}$ |
| (S) | If $\sin^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \log a$, then $\frac{dy}{dx}$ is equal to | (4) | $-1$ |

(a) P-2, Q-1, R-4, S-2 (b) P-3, Q-4, R-1, S-2

(c) P-2, Q-2, R-4, S-3 (d) P-3, Q-1, R-4, S-2

**40.**

| | Column I | | Column II |
|---|---|---|---|
| (P) | The slope of the curve $2y^2 = ax^2 + b$ at $(1, -1)$ is $-1$, then | (1) | $a - b = 2$ |
| (Q) | If $(a, b)$ be the point on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes, then | (2) | $a - b = 7/2$ |
| (R) | If the tangent at any point (1, 2) on the curve $y = ax^2 + bx + 7/2$ be parallel to the normal at $(-2, 2)$ on the curve $y = x^2 + 6x + 10$, then | (3) | $a - b = 4/3$ |
| | | (4) | $a + \lvert b\rvert = \frac{20}{3}$ |

(a) P-1; Q-3,4; R-2 (b) P-2; Q-3,4; R-1

(c) P-3; Q-1; R-2, 4 (d) P-4; Q-3; R-1, 2

## Case Based MCQs

**Case I :** Read the following passage and answer the questions from 41 to 45.

If a real valued function $f(x)$ is finitely derivable at any point of its domain, it is necessarily continuous at that point. But its converse need not be true.

For example, every polynomial, constant function are both continuous as well as differentiable and inverse trigonometric functions are continuous and differentiable in its domain etc.

**41.** If $f(x) = \begin{cases} x, \text{ for } x \le 0 \\ 0, \text{for } x > 0 \end{cases}$, then at $x = 0$

(a) $f(x)$ is differentiable and continuous

(b) $f(x)$ is neither continuous nor differentiable

(c) $f(x)$ is continuous but not differentiable

(d) none of these

**42.** If $f(x) = \lvert x - 9\rvert$, $x \in R$, then at $x = 9$

(a) $f(x)$ is not continuous

(b) $f(x)$ is continuous but not differentiable

(c) $f(x)$ is continuous and differentiable

(d) None of these

**43.** $f(x) = x^3$ is

(a) continuous but not differentiable at $x = 7$

(b) continuous and differentiable at $x = 7$

(c) neither continuous nor differentiable at $x = 7$

(d) None of these

**44.** If $f(x) = [\cos x]$, then which of the following is true?

(a) $f(x)$ is continuous and differentiable at $x = 0$.

(b) $f(x)$ is discontinuous at $x = 0$.

(c) $f(x)$ is continuous at $x = 0$ but not differentiable

(d) None of these

**45.** If $f(x) = \sin^{-1} x$, $-1 \le x \le 1$, then

(a) $f(x)$ is both continuous and differentiable

(b) $f(x)$ is neither continuous nor differentiable

(c) $f(x)$ is continuous but not differentiable

(d) None of these

**Case II :** Read the following passage and answer the questions from 46 to 50.

An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 32000 $l$ of water, that comes out from a tube well.

**46.** If the length, width and height of the open tank be $x$, $x$ and $y$ m respectively, then total surface area of tank is

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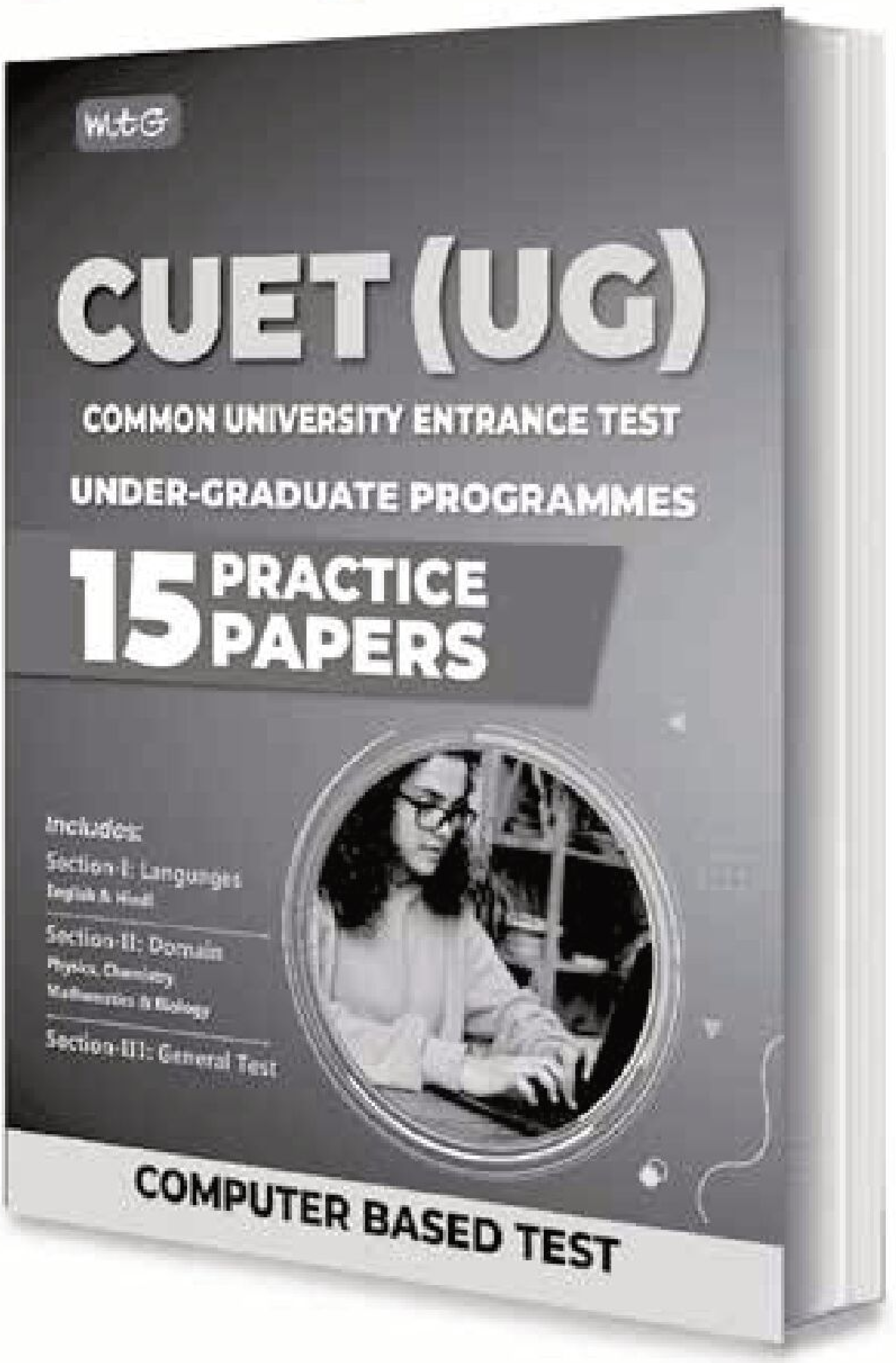
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(a) $(x^2 + 2xy)\ \text{m}^2$ (b) $(2x^2 + 4xy)\ \text{m}^2$
(c) $(2x^2 + 2xy)\ \text{m}^2$ (d) $(2x^2 + 8xy)\ \text{m}^2$

**47.** The relation between $x$ and $y$ is
(a) $x^2y = 32$ (b) $xy^2 = 32$ (c) $x^2y^2 = 32$ (d) $xy = 32$

**48.** The outer surface area of tank will be minimum when depth of tank is equal to
(a) half of its width (b) its width
(c) $\left(\frac{1}{4}\right)^{\text{th}}$ of its width (d) $\left(\frac{1}{3}\right)^{\text{rd}}$ of its width

**49.** The cost of material will be least when width of tank is equal to
(a) half of its depth (b) twice of its depth
(c) $\left(\frac{1}{4}\right)^{\text{th}}$ of its depth (d) thrice of its depth

**50.** If cost of aluminium sheet is ₹ 360/m$^2$, then the minimum cost for the construction of tank will be
(a) ₹ 15,000 (b) ₹ 16280
(c) ₹ 17280 (d) ₹ 18280

## SOLUTIONS

**1. (c) :** L.H.L. $= \lim\limits_{x\to 0} \dfrac{\sin(a+1)x + \sin x}{x} \left(\dfrac{0}{0} \text{ form}\right)$

Using L′ Hospital Rule, we get

L.H.L. $= \lim\limits_{x\to 0} (a+1)\cos(a+1)x + \cos x = a+2$ ...(i)

R.H.L. $= \lim\limits_{x\to 0} \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} = \lim\limits_{x\to 0} \dfrac{\sqrt{1+bx}-1}{bx}$

$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{1+bx}+1} = \dfrac{1}{2}$ ...(ii)

$f(0) = C$ ...(iii)

From (i), (ii) and (iii), we get

$a+2 = c = \dfrac{1}{2} \Rightarrow a = -\dfrac{3}{2}$ and $a+c = -1$

**2. (c) :** Given, $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ ...(i)

Squaring both sides, we get $x^2(1 + y) = y^2 (1 + x)$

$\Rightarrow (x - y)(x + y + xy) = 0 \Rightarrow y = x$ or $y = -\dfrac{x}{1+x}$

$\therefore \dfrac{dy}{dx} = \dfrac{-(1+x)\cdot 1 + x\cdot 1}{(1+x)^2} = \dfrac{-1}{(1+x)^2}$

**3. (b):** $f(0) = \lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{x}$

$= \lim\limits_{x\to 0}\left(\dfrac{\sqrt{4+x}-2}{x} \times \dfrac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right) = \lim\limits_{x\to 0} \dfrac{4+x-4}{x(\sqrt{4+x}+2)}$

$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{4+x}+2} = \dfrac{1}{2+2} = \dfrac{1}{4}.$

**4. (a) :** Given, $x\sin(a + y) = \sin y \Rightarrow x = \dfrac{\sin y}{\sin(a+y)}$

Differentiating both sides w.r.t. $x$, we get

$$1 = \frac{\sin(a+y)\cos y \dfrac{dy}{dx} - \cos(a+y)\sin y \dfrac{dy}{dx}}{\sin^2(a+y)}$$

$\Rightarrow \sin^2(a + y) = \sin(a + y - y)\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$

**5. (d) :** $|x-3|$ is continuous at $x = 3$, but not differentiable.

$f(1^-) = f(1^+) = f(1) = 2$

$f'(1^-) = f'(1^+) = -1 = f'(1)$

$\therefore$ $f(x)$ is continuous and differentiable at $x = 1$.

**6. (a) :** We have, $\sqrt{x+y} + \sqrt{y-x} = a$

Differentiating w.r.t. $x$, we get

$$\frac{1}{2\sqrt{x+y}}\left(1+\frac{dy}{dx}\right) + \frac{1}{2\sqrt{y-x}}\left(\frac{dy}{dx}-1\right) = 0$$

$$\Rightarrow \frac{-\left(\sqrt{y-x}-\sqrt{x+y}\right)}{\left(\sqrt{y-x}+\sqrt{x+y}\right)} = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{x+y}-\sqrt{y-x}}{\sqrt{y-x}+\sqrt{x+y}}$$

**7. (d):** Given point is $x = a + bt - ct^2$, $y = at + bt^2$

Acceleration in $x$-direction $= \dfrac{d^2x}{dt^2} = -2c$

and acceleration in $y$-direction $= \dfrac{d^2y}{dt^2} = 2b$

$\therefore$ Resultant acceleration $= \sqrt{\left(\dfrac{d^2x}{dt^2}\right)^2 + \left(\dfrac{d^2y}{dt^2}\right)^2}$

$= \sqrt{(-2c)^2 + (2b)^2} = 2\sqrt{b^2+c^2}$

**8. (c) :** The given curve is $y^2e^{xy} = 9e^{-3}x^2$

Taking log on both sides, we get

$2\log y + xy = \log 9 - 3 + 2\log x$ ...(i)

Differentiating (i) w.r.t '$x$', we get

$$\frac{2}{y}\frac{dy}{dx} + x\frac{dy}{dx} + y = \frac{2}{x} \Rightarrow \frac{dy}{dx} = \frac{\dfrac{2}{x} - y}{\dfrac{2}{y} + x}$$

Now, $\left(\dfrac{dy}{dx}\right)_{(-1,\,3)} = \dfrac{\dfrac{2}{-1} - 3}{\dfrac{2}{3} - 1} = \dfrac{-5\times 3}{-1} = 15$

**9. (d)**

**10. (b) :** Since, $\frac{dy}{dx} = \cos x$, therefore, slope of tangent at (0, 0) = cos 0 = 1 and hence slope of normal at (0, 0) is – 1.

$\therefore$ Equation of normal is $y - 0 = -1(x - 0) \Rightarrow x + y = 0$

**11. (d) :** $f(x) = (1 + x)^n = (x + 1)^n$

$f(x) = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n$

$f'(x) = {}^nC_1 + {}^nC_2(2x) + \dots. + {}^nC_n(nx^{n-1})$

$f''(x) = 2\,{}^nC_2 + 6\,{}^nC_3x + \dots. + {}^nC_n(n(n-1)x^{n-2})$

$f^n(x) = {}^nC_n\,(n)(n-1)\,\dots\dots\dots\, 3.2.1.$

$$f(0) + \frac{f'(0)}{1!} + \frac{f''(0)}{2!} + \dots + \frac{f^n(0)}{n!}$$
$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

**12. (b) :** Given $f(x) = x^{\alpha}\log x$ and $f(0) = 0$

Also, $f(1) = 0 \Rightarrow f(1) = f(0)$

Since, given function is continuous on [0, 1] and derivable on (0, 1).

$\therefore$ There exists $c \in (0, 1)$ such that $f'(c) = 0$

Now, $f'(x) = \frac{x^{\alpha}}{x} + (\alpha \log x)(x)^{\alpha-1} = x^{\alpha-1}(1 + \alpha \log x)$

$\because$ $f'(c) = 0 \Rightarrow c^{\alpha-1}(1 + \alpha\log c) = 0$

$\Rightarrow$ $1 + \alpha\log c = 0$ $\quad [\because c \in (0, 1)]$

$\Rightarrow$ $\alpha = \frac{-1}{\log c}$

Since, we know, $\log c < 0$, when $c \in (0, 1)$ and $\log c \geq 0$ when $c \geq 1$.

**13. (a) :** Let $y = \sin^p\theta\cos^q\theta$

$$\Rightarrow \frac{dy}{d\theta} = p\sin^{p-1}\theta\cos\theta\cos^q\theta + \sin^p\theta\, q\cos^{q-1}\theta(-\sin\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = p\sin^{p-1}\theta\,\cos^{q+1}\theta - q\sin^{p+1}\theta\cos^{q-1}\theta$$

$$\Rightarrow \frac{dy}{d\theta} = \sin^{p-1}\theta\,\cos^{q-1}\theta(p\cos^2\theta - q\sin^2\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = \sin^p\theta\,\cos^q\theta\left(\frac{p\cos^2\theta - q\sin^2\theta}{\sin\theta\cos\theta}\right)$$

$$\Rightarrow \frac{dy}{d\theta} = \sin^p\theta\,\cos^q\theta(p\cot\theta - q\tan\theta)$$

For maximum or minimum, we have

$\frac{dy}{d\theta} = 0 \Rightarrow \sin^p\theta\cos^q\theta\,(p\cot\theta - q\tan\theta) = 0$

$\Rightarrow$ $\sin^p\theta = 0$ or $\cos^q\theta = 0$ or $p\cot\theta - q\tan\theta = 0$

$\Rightarrow$ $\sin^p\theta = 0$ or $\cos^q\theta = 0$ or $\tan\theta = \sqrt{\frac{p}{q}}$

$\Rightarrow$ $\theta = 0$ or $\theta = \frac{\pi}{2}$ or $\theta = \tan^{-1}\sqrt{\frac{p}{q}} = \alpha$ (say)

Now, $\frac{dy}{d\theta} = \sin^p\theta\cos^q\theta(p\cot\theta - q\tan\theta)$
$= y(p\cot\theta - q\tan\theta)$

$$\Rightarrow \frac{d^2y}{d\theta^2} = \frac{dy}{d\theta}(p\cot\theta - q\tan\theta) + y(-p\,\mathrm{cosec}^2\theta - q\sec^2\theta)$$

$$\Rightarrow \left(\frac{d^2y}{d\theta^2}\right)_{\theta=\alpha} = \left(\frac{dy}{d\theta}\right)_{\theta=\alpha}\left(p\sqrt{\frac{q}{p}} - q\sqrt{\frac{p}{q}}\right)$$
$$+ \sin^p\theta\,\cos^q\theta[-p\,\mathrm{cosec}^2\theta - q\sec^2\theta]$$

$$\Rightarrow \left(\frac{d^2y}{d\theta^2}\right)_{\theta=\alpha} = 0 - \sin^p\theta\cos^q\theta(p\,\mathrm{cosec}^2\theta + q\sec^2\theta) < 0$$

Hence, $y$ is maximum when $\theta = \alpha = \tan^{-1}\sqrt{\frac{p}{q}}$.

**14. (a) :** Here, $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$

$\Rightarrow$ $f'(x) = 3x^2 + 12x + 9 + 2k$

Now $f$ is strictly increasing for all $x \in R$

if $f'(x) > 0\ \forall\ x \in R$

$\Rightarrow$ $3x^2 + 12x + (9 + 2k) > 0$ for all $x \in R$

$\Rightarrow 12^2 - 4{\cdot}3\,(9 + 2k) < 0 \Rightarrow 12 - 9 - 2k < 0 \Rightarrow 3 - 2k < 0$

$\Rightarrow$ $k > 3/2$

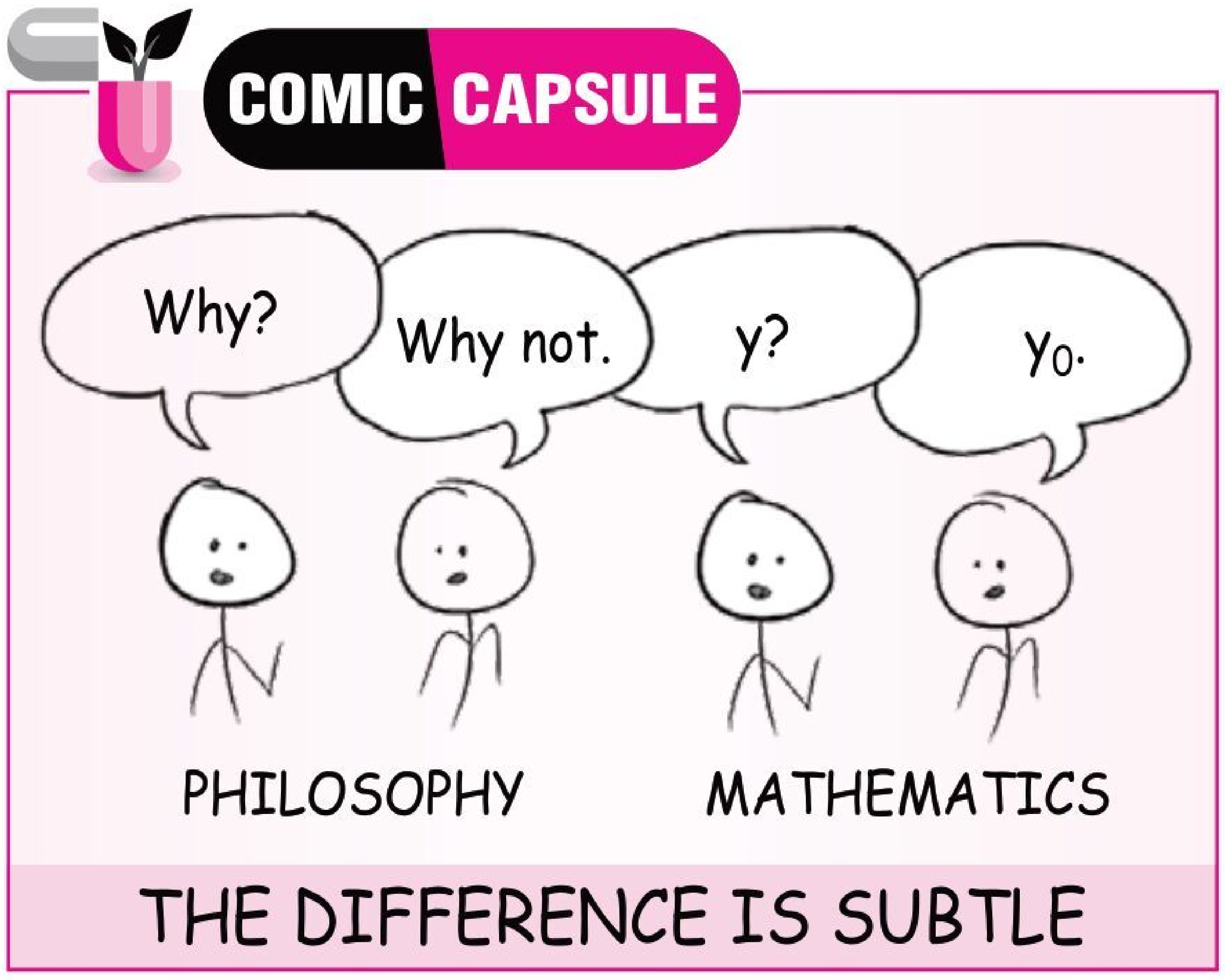
**15. (b) :** Applying Mean Value theorem to $f(x)$ in $[0, x]$, $0 < x \leq 2$, we get $\frac{f(x) - f(0)}{x - 0} = f'(c)$ for some $c \in (0, 2)$

$\Rightarrow$ $\frac{f(x)}{x} = f'(c) \leq \frac{1}{2} \Rightarrow f(x) \leq \frac{1}{2}x \leq \frac{1}{2}\cdot 2$ $\quad (\because x \leq 2)$

$\Rightarrow$ $f(x) \leq 1$

**16. (a) :** Let edge of cube be $x$ cm

$\therefore$ $\frac{\Delta x}{x} \times 100 = a$ (Given)



Now, $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$

So, $\Delta S = \frac{dS}{dx}\times\Delta x \Rightarrow \frac{\Delta S}{S}\times 100 = \frac{12x}{6x^2}\times\frac{ax}{100}\times 100 = 2a\%$

**17. (c)**

**18. (d) :** Let $V$ be the velocity of a particle at time $t$.

We have, $S \propto \sqrt[3]{V} \Rightarrow KS^3 = V$

$\Rightarrow \frac{dS}{dt} = V = KS^3$ (where $K$ is any constant) ...(i)

$\Rightarrow \frac{d^2S}{dt^2} = 3KS^2\frac{dS}{dt} = 3KS^2(KS^3)$ (From (i))

$= 3K^2S^5$. So, acceleration $\propto S^5$

**19. (c) :** We have, $\sin y + e^{-x\cos y} = e$

Differentiating both sides w.r.t. $x$, we get

$$\cos y\frac{dy}{dx} + e^{-x\cos y}\left\{x\sin y\frac{dy}{dx} - \cos y\right\} = 0$$

$\Rightarrow (\cos y + e^{-x\cos y}\cdot x\sin y)\frac{dy}{dx} = (e^{-x\cos y})\cos y$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{-x\cos y}\cdot\cos y}{\cos y + x\cdot e^{-x\cos y}\sin y}$$

$$\therefore \left[\frac{dy}{dx}\right]_{(1,\pi)} = \frac{e^{-\cos\pi}\cdot\cos\pi}{\cos\pi + e^{-\cos\pi}\sin\pi} = e$$

**20. (b)**

**21. (d) :** $f(x) = \frac{(1+x)^{3/5}}{1+x^{3/5}}$; When $f'(x) = 0 \Rightarrow x = 1$

$f(x)$ is strictly decreasing function

Also, $f(0) = 1$, and $f(1) = \frac{2^{0.6}}{2} = 2^{-0.4}$

$\therefore f(x) \in [2^{-0.4}, 1]$

**22. (d) :** Given, $f(x) = 1 - x^3 - x^5$

Differentiating w.r.t. $x$, we get $f'(x) = -3x^2 - 5x^4$

$\Rightarrow f'(x) = -(3x^2 + 5x^4) \Rightarrow f'(x) \le 0$ for all values of $x$.

**23. (d) :** Given that, $\frac{dV}{dt} = k$ (say)

$\because V = \frac{4}{3}\pi R^3 \Rightarrow \frac{dV}{dt} = 4\pi R^2\frac{dR}{dt} \Rightarrow \frac{dR}{dt} = \frac{k}{4\pi R^2}$

⇒ Rate of increase of radius is inversely proportional to its surface area.

**24. (c) :** $y^2 = x(2-x)^2 \Rightarrow y^2 = x^3 - 4x^2 + 4x$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 3x^2 - 8x + 4 \Rightarrow \frac{dy}{dx} = \frac{3x^2 - 8x + 4}{2y}$

$\Rightarrow \left[\frac{dy}{dx}\right]_P = \frac{3-8+4}{2} = -\frac{1}{2}$

∴ Equation of tangent at $P$ is

$y - 1 = -\frac{1}{2}(x-1) \Rightarrow x + 2y - 3 = 0$ ...(ii)

Using $y = \frac{3-x}{2}$ in (i), we get $\left(\frac{3-x}{2}\right)^2 = x^3 - 4x^2 + 4x$

$\Rightarrow 9 + x^2 - 6x = 4x^3 - 16x^2 + 16x$

$\Rightarrow 4x^3 - 17x^2 + 22x - 9 = 0$,

which has two roots 1, 1 (Because of (ii) being tangent at (1, 1)).

Sum of 3 roots $= \frac{17}{4} \Rightarrow 3^{\text{rd}}$ root $= \frac{17}{4} - 2 = \frac{9}{4}$

Then, $y = \frac{3 - \frac{9}{4}}{2} = \frac{3}{8}$ $\therefore$ $Q$ is $\left(\frac{9}{4}, \frac{3}{8}\right)$.

**25. (d) :** $y^2 = 100\cdot\frac{\pi}{2} + 45\cdot\frac{\pi}{2} = 145\frac{\pi}{2}$

$\left(\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, \sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}\right)$

$\Rightarrow 2yy' = 0 \Rightarrow y' = 0$ $(\because y \ne 0)$

**26. (c) :** $y = \tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log ex^2}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

$= \tan^{-1}\left(\frac{1-\log x^2}{1+\log x^2}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

$= \tan^{-1}(1) - \tan^{-1}(2\log x) + \tan^{-1}(3) + \tan^{-1}(2\log x)$

$\therefore y = \tan^{-1}(1) + \tan^{-1}(3) \Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$

**27. (a) :** Let $g(x) = ax^3 + bx^2 + cx$

Now $g(0) = 0$ and $g(1) = a + b + c = 0$

As $g(0)$ and $g(1)$ are zeroes then $g'(x)$ will have at least one root between 0 and 1.

**28. (b) :** Given, $f(x) = x + \frac{4}{x}$

$f'(x) = 1 - \frac{4}{x^2}$ $\therefore f'(x) = 0 \Rightarrow x = \pm 2$

and $f''(x) = \frac{8}{x^3} > 0$ which is $> 0$ for $x = 2$ and $< 0$ for $x = -2$

$\therefore f(x)$ has local minima at $x = 2$ and local maxima at $x = -2$.

## MONTHLY TEST DRIVE CLASS XI ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (a) | **2.** | (a) | **3.** | (c) | **4.** | (a) | **5.** | (a) |
| **6.** | (c) | **7.** | (a,c,d) | **8.** | (c) | **9.** | (a, c) | **10.** | (c) |
| **11.** | (b, d) | **12.** | (b) | **13.** | (c) | **14.** | (b) | **15.** | (b) |
| **16.** | (b) | **17.** | (60) | **18.** | (3) | **19.** | (89) | **20.** | (70) |

**29. (d):** Let $P = xy^3$ ...(i)

Given, $x + y = 60 \Rightarrow x = 60 - y$

Putting value of $x$ in (i), we get

$$P = (60 - y)y^3 = 60y^3 - y^4$$

Differentiating w.r.t. $y$, we get

$$\frac{dP}{dy} = 180y^2 - 4y^3 \Rightarrow \frac{d^2P}{dy^2} = 360y - 12y^2$$

For maxima or minimum, we must have $\frac{dP}{dy} = 0$

$\therefore \quad 180y^2 - 4y^3 = 0 \Rightarrow 4y^2(45 - y) = 0 \Rightarrow y = 0, 45$

But $y \neq 0$, so $y = 45$

$$\therefore \quad \left(\frac{d^2P}{dy^2}\right)_{y=45} = 360 \times 45 - 12 \times (45)^2$$

$$= 16200 - 24300 = -8100 < 0$$

$\Rightarrow$ $P$ has a local maxima at $y = 45$.

By second derivative test, $y = 45$ is a point of local maxima of $P$. Thus, function $xy^3$ is maximum when $y = 45$ and $x = 60 - 45 = 15$.

**30. (c) :** $\frac{d}{dx}\left\{\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)\right\} = \frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}$

$$= \frac{1+x^2}{\sqrt{(1-x^2)^2}} \cdot \frac{(1+x^2)2 - 2x \cdot 2x}{(1+x^2)^2} = \frac{2(1-x^2)}{(1+x^2)|1-x^2|}$$

**31. (d):** $\because f(x) = \sin x - \cos x$

Differentiating w.r.t. $x$, we get $f'(x) = \cos x + \sin x$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x\right) = \sqrt{2}\left[\cos\left(x - \frac{\pi}{4}\right)\right]$$

For decreasing $f'(x) < 0$

$$\frac{\pi}{2} < \left(x - \frac{\pi}{4}\right) < \frac{3\pi}{2} \qquad (\text{within } 0 \le x \le 2\pi)$$

$$\Rightarrow \frac{\pi}{2} + \frac{\pi}{4} < \left(x - \frac{\pi}{4} + \frac{\pi}{4}\right) < \frac{3\pi}{2} + \frac{\pi}{4} \Rightarrow \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

**32 (b):** $\frac{f'(c)}{g'(c)} = \frac{f(1) - f(0)}{g(1) - g(0)} \Rightarrow 2 = \frac{6-2}{g(1) - 0} \Rightarrow g(1) = 2$

**33. (a) :** Here, $\frac{dy}{dx} = 3x^2 + 3 \Rightarrow$ slopes of tangents at $x = 1$ and $x = -1$ are equal. Hence, the two tangents are parallel.

**34. (b):** Let $(\alpha, \beta)$ be the required point, then $\beta^2 = \alpha$

and $\left(\frac{dy}{dx}\right)_{(\alpha,\beta)} = \tan 45° = 1 \Rightarrow \frac{1}{2\beta} = 1 \Rightarrow \beta = \frac{1}{2}$

Also, $\alpha = \beta^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Hence, the required point is $\left(\frac{1}{4}, \frac{1}{2}\right)$.

**35. (c) :** Given, $f(x) = x^2\sqrt{1+x}$ ...(i)

The given function is differentiable for all $x$ in $\left[-1, \frac{1}{2}\right]$.

Differentiating (i) w.r.t. $x$, we get

$$f'(x) = x^2 \cdot \frac{1}{2}(1+x)^{-1/2} + \sqrt{1+x} \cdot 2x$$

$$= \frac{x^2}{2\sqrt{1+x}} + 2x\sqrt{1+x} = \frac{5x^2 + 4x}{2\sqrt{1+x}}.$$

Now $f'(x) = 0 \Rightarrow \frac{5x^2 + 4x}{2\sqrt{1+x}} = 0 \Rightarrow 5x^2 + 4x = 0$

$$\Rightarrow \quad x(5x + 4) = 0 \Rightarrow x = 0, -\frac{4}{5}$$

Also $0, -\frac{4}{5}$ both lie in $\left[-1, \frac{1}{2}\right]$, therefore, 0 and $-\frac{4}{5}$ both are stationary points.

Further, $f(0) = 0, f\left(-\frac{4}{5}\right) = \frac{16}{25} \cdot \frac{1}{\sqrt{5}} = \frac{16}{25\sqrt{5}}$

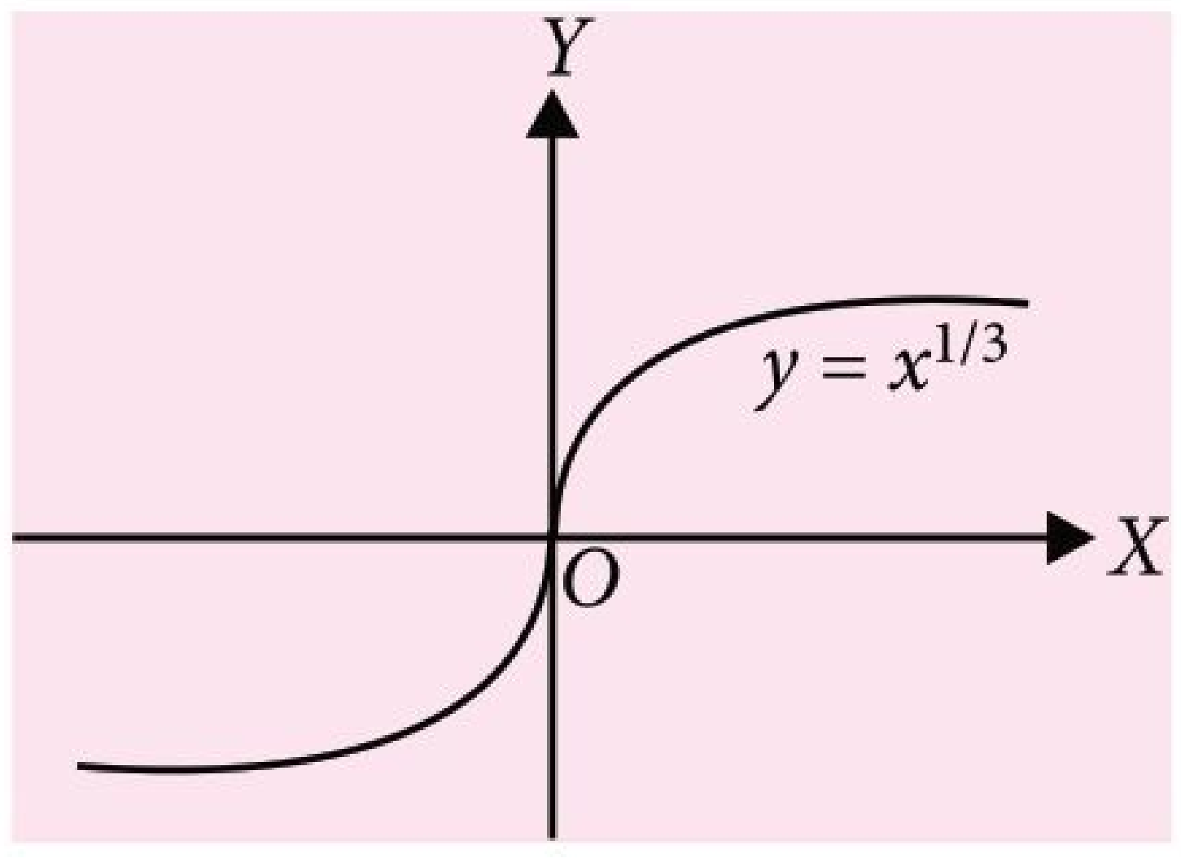
$$f(-1) = 0, f\left(\frac{1}{2}\right) = \frac{1}{4} \cdot \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{8}$$

Therefore, the absolute maximum value is $\frac{\sqrt{6}}{8}$ and the absolute minimum value is 0.

**36. (d):** Given $f(x) = x^{1/3}, x \in R$

$$\Rightarrow \quad f'(x) = \frac{1}{3}x^{-2/3}$$

$$\text{and } \quad f''(x) = \frac{1}{3}\left(-\frac{2}{3}\right)x^{-5/3} = \frac{-2}{9x^{5/3}}$$



Both $f'(x)$ and $f''(x)$ exist at all points except at $x = 0$

So, $f''(0)$ does not exist.

However, we note that $f''(x) > 0$ for $x < 0$ and $f''(x) < 0$ for $x > 0$

$\Rightarrow$ $f''(x)$ changes sign from positive to negative as we move from left to right through 0.

So, $f$ has a point of inflexion at $x = 0$.

**37.** **(a) :** We have, $y=\cot^{-1}\left(\frac{1+x\sqrt{x}}{\sqrt{x}-x}\right)$

Let $x=\cot\phi$ and $\sqrt{x}=\cot\theta$

$$y=\cot^{-1}\left(\frac{1+\cot\theta\cot\phi}{\cot\theta-\cot\phi}\right)=\phi-\theta=\cot^{-1}x-\cot^{-1}\sqrt{x}$$

$$\therefore\quad \frac{dy}{dx}=\frac{-1}{1+x^2}+\frac{1}{1+x}\times\frac{1}{2\sqrt{x}}$$

**38.** **(c)**

**39.** **(d) : (P)** $y=\log_e|2x| \Rightarrow y=\log_e(|2||x|)$

$\Rightarrow\ y=\log_e|x|+C$ [where $C=\log_e 2$]

For $x\ge 0$, $y=\log_e x+C$ [$\because |x|=x$ for $x\ge 0$]

$\Rightarrow\ \frac{dy}{dx}=\frac{1}{x}$ ...(i)

For $x<0$, $y=\log_e(-x)+C$ [$\because |x|=-x$ for $x<0$]

$\Rightarrow\ \frac{dy}{dx}=\frac{1}{-x}\times(-1)=\frac{1}{x}$ ...(ii)

Combining (i) and (ii) we have, $\frac{dy}{dx}=\frac{1}{x}$

**(Q)** Let $y=\log_{10}x$ and $z=x^2$

Taking derivative w.r.t. $x$, we get

$\frac{dy}{dx}=\frac{1}{x\log_e 10}$ ...(i) $\quad \frac{dz}{dx}=2x$ ...(ii)

$$\therefore\quad \frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\frac{1}{x\times 2x\log_e 10}=\frac{\log_{10}e}{2x^2}$$

**(R)** Given $x^y y^x=16$

$\Rightarrow\ \log(x^y y^x)=\log 16 \Rightarrow\ \log x^y+\log y^x=\log 16$

$\Rightarrow\ y\log x+x\log y=\log 16$

Differentiating both sides, w.r.t. $x$, we get

$$y\cdot\frac{1}{x}+\frac{dy}{dx}\cdot\log x+x\cdot\frac{1}{y}\cdot\frac{dy}{dx}+1\cdot\log y=0$$

$$\Rightarrow\ \frac{dy}{dx}\left(\log x+\frac{x}{y}\right)=\frac{-y}{x}-\log y$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(y/x+\log y)}{(\log x+x/y)} \Rightarrow \left(\frac{dy}{dx}\right)_{(2,2)}=-\left(\frac{1+\log 2}{\log 2+1}\right)=-1$$

**(S)** We have, $\frac{x^2-y^2}{x^2+y^2}=\sin(\log a)=$ constant $=k$ (say)

$\Rightarrow\ x^2-y^2=kx^2+ky^2$

$\Rightarrow\ (k+1)y^2=(1-k)x^2 \Rightarrow y^2=\frac{1-k}{1+k}x^2$ ...(i)

Differentiating both sides w.r.t. $x$, we get

$$2y\frac{dy}{dx}=\frac{1-k}{1+k}\cdot 2x \ \Rightarrow\ \frac{dy}{dx}=\frac{1-k}{1+k}\cdot\frac{x}{y}=\frac{y^2}{x^2}\cdot\frac{x}{y} \text{ [From (i)]}$$

$$\therefore\quad \frac{dy}{dx}=\frac{y}{x}.$$

**40.** **(a) : (P)** Here, $2y^2=ax^2+b$

$\Rightarrow\ 4y\frac{dy}{dx}=2ax \Rightarrow \left[\frac{dy}{dx}\right]_{(1,-1)}=\frac{a}{-2}=-1$ [Given]

$\Rightarrow\ a=2$

Also, $(1,-1)$ lies on $2y^2=ax^2+b$

$\Rightarrow\ 2=a+b \Rightarrow b=0 \Rightarrow a-b=2$

**(Q)** $\because (a,b)$ lies on the curve $9y^2=x^3$

$\therefore\ 9b^2=a^3$ ...(i)

Now, $18y\frac{dy}{dx}=3x^2$ and $\frac{dy}{dx}=\frac{x^2}{6y} \Rightarrow \left[\frac{dy}{dx}\right]_{(a,b)}=\frac{a^2}{6b}$

Since, the normal to the curve at $(a,b)$ makes equal intercepts with the coordinate axes. Therefore, slope of normal $=\pm 1$.

$$\Rightarrow\ -\frac{1}{\left[\frac{dy}{dx}\right]_{(a,b)}}=\pm 1 \Rightarrow \left[\frac{dy}{dx}\right]_{(a,b)}=\pm 1 \Rightarrow \frac{a^2}{6b}=\pm 1$$

$\Rightarrow\ a^2=\pm 6b \ \Rightarrow\ a^4=36b^2=36\left(\frac{a^3}{9}\right)$ [From (i)]

$\therefore\ a=0, 4 \Rightarrow b=0, \pm 8/3.$

But the line making equal intercepts with the coordinate axes can not pass through the origin.

Hence, the required points are $(4, 8/3)$ and $(4, -8/3)$

$\therefore\ a-b=4/3$ and $a-b=20/3$

and $a+|b|=4+8/3=20/3$

**(R)** We have, $y=ax^2+bx+7/2 \Rightarrow \frac{dy}{dx}=2ax+b$

$\because$ $(1, 2)$ lies on $y=ax^2+bx+7/2$

$\therefore\ 2=a+b+7/2 \Rightarrow a+b+3/2=0$ ....(i)

$\left[\frac{dy}{dx}\right]_{(1,2)}=2a(1)+b=2a+b$ ...(ii)

Also, we have $y=x^2+6x+10 \Rightarrow \frac{dy}{dx}=2x+6$

$$\therefore\quad -\frac{1}{\left[\frac{dy}{dx}\right]_{(-2,2)}}=-\frac{1}{2(-2)+6}=-\frac{1}{2}$$ ....(iii)

Given, $2a+b=-\frac{1}{2}$ ...(iv)

Solving (i) and (iv), we get

$a=1, b=-5/2 \Rightarrow a-b=7/2$

**41.** **(c)** **42.** **(b)** **43.** **(b)** **44.** **(b)**
**45.** **(a)** **46.** **(d)** **47.** **(a)** **48.** **(a)**
**49.** **(b)** **50.** **(c)**

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# NDA & NA Exam (II)

## National Defence Academy and Naval Academy Examination II

# SOLVED PAPER 2022

## MATHEMATICS

**1.** How many four-digit natural numbers are there such that all of the digits are odd?

(a) 625 (b) 400 (c) 196 (d) 120

**2.** What is $\sum_{r=0}^{n} 2^r C(n,r)$ equal to?

(a) $2^n$ (b) $3^n$ (c) $2^{2n}$ (d) $3^{2n}$

**3.** If different permutations of the letters of the word 'MATHEMATICS' are listed as in a dictionary, how many words (with or without meaning) are there in the list before the first word that starts with C ?

(a) 302400 (b) 403600 (c) 907200 (d) 1814400

**4.** Consider the following statements :

1. If $f$ is the subset of $Z \times Z$ defined by $f = \{(xy, x-y);\ x,y \in Z\}$, then $f$ is a function from $Z$ to $Z$.
2. If $f$ is the subset of $N \times N$ defined by $f = \{(xy, x + y);\ x, y \in N\}$, then $f$ is a function from $N$ to $N$.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**5.** Consider the determinant $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$.

If $a_{13} = yz$, $a_{23} = zx$, $a_{33} = xy$ and the minors of $a_{13}, a_{23}, a_{33}$ are respectively $(z - y), (z - x), (y - x)$, then what is the value of $\Delta$ ?

(a) $(z - y)(z - x)(y - x)$
(b) $(x - y)(y - z)(x - z)$
(c) $(x - y)(z - x)(y - z)(x + y + z)$
(d) $(xy + yz + zx)(x + y + z)$

**6.** If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{pmatrix}$, then which of the following are correct ?

1. $A + adjA$ is a null matrix
2. $A^{-1} + adjA$ is a null matrix
3. $A - A^{-1}$ is a null matrix

Select the correct answer using the code given below :

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

**7.** If $X$ is a matrix of order $3 \times 3$, $Y$ is a matrix of order $2 \times 3$ and $Z$ is a matrix of order $3 \times 2$, then which of the following are correct ?

1. $(ZY)X$ is a square matrix having 9 entries.
2. $Y(XZ)$ is a square matrix having 4 entries.
3. $X(YZ)$ is not defined.

Select the correct answer using the code given below :

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

**8.** For how many quadratic equations, the sum of roots is equal to the product of roots?

(a) 0 (b) 1
(c) 2 (d) Infinitely many

**9.** Consider the following statements :

1. The set of all irrational numbers between $\sqrt{2}$ and $\sqrt{5}$ is an infinite set.
2. The set of all odd integers less than 100 is a finite set.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**10.** Consider the following statements :

1. $2 + 4 + 6 + ... + 2n = n^2 + n$
2. The expression $n^2 + n + 41$ always gives a prime number for every natural number $n$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**11.** Let $p, q$ $(p > q)$ be the roots of the quadratic equation $x^2 + bx + c = 0$ where $c > 0$. If $p^2 + q^2 - 11pq = 0$, then what is $p - q$ equal to ?

(a) $3\sqrt{c}$ (b) $3c$ (c) $9\sqrt{c}$ (d) $9c$

**12.** What is the diameter of a circle inscribed in a regular polygon of 12 sides, each of length 1 cm ?

(a) $1+\sqrt{2}$ cm (b) $2+\sqrt{2}$ cm

(c) $2+\sqrt{3}$ cm (d) $3+\sqrt{3}$ cm

**13.** Let $A = \{7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$ and let $f : A \to N$ be defined by $f(x)$ = the highest prime factor of $x$. How many elements are there in the range of $f$?

(a) 4 (b) 5 (c) 6 (d) 7

**14.** Let $R$ be a relation from $N$ to $N$ defined by $R = \{(x, y) : x, y \in N \text{ and } x^2 = y^3\}$. Which of the following are not correct?

1. $(x, x) \in \mathrm{R}$ for all $x \in \mathrm{N}$
2. $(x, y) \in \mathrm{R} \Rightarrow (y, x) \in \mathrm{R}$
3. $(x, y) \in \mathrm{R}$ and $(y, z) \in \mathrm{R} \Rightarrow (x, z) \in \mathrm{R}$

Select the correct answer using the code given below :

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

**15.** Consider the following :

1. $A \cap B = A \cap C \Rightarrow B = C$
2. $A \cup B = A \cup C \Rightarrow B = C$

Which of the above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Consider the following for the next ***three*** **(03)** items that follow :

Let $z = \dfrac{1+i\sin\theta}{1-i\sin\theta}$ where $i = \sqrt{-1}$

**16.** What is the modulus of $z$ ?

(a) 1 (b) $\sqrt{2}$

(c) $1 + \sin^2\theta$ (d) $\dfrac{1+\sin^2\theta}{1-\sin^2\theta}$

**17.** What is angle $\theta$ such that $z$ is purely real?

(a) $\dfrac{n\pi}{2}$ (b) $\dfrac{(2n+1)\pi}{2}$

(c) $n\pi$ (d) $2n\pi$ only

where $n$ is an integer

**18.** What is angle $\theta$ such that z is purely imaginary?

(a) $\dfrac{n\pi}{2}$ (b) $\dfrac{(2n+1)\pi}{2}$

(c) $n\pi$ (d) $2n\pi$

where $n$ is an integer

Consider the following for the next ***three*** **(03)** items that follow :

Let $P$ be the sum of first $n$ positive terms of an increasing arithmetic progression $A$. Let $Q$ be the sum of first $n$ positive terms of another increasing arithmetic progression $B$.

Let $P : Q = (5n + 4) : (9n + 6)$

**19.** What is the ratio of the first term of $A$ to that of $B$?

(a) 1/3 (b) 2/5 (c) 3/4 (d) 3/5

**20.** What is the ratio of their 10th terms ?

(a) 11/29 (b) 22/49 (c) 33/59 (d) 44/69

**21.** If $d$ is the common difference of $A$, and $D$ is the common difference of $B$, then which one of the following is always correct ?

(a) $D > d$ (b) $D < d$

(c) $7D > 12d$ (d) None of the above

Consider the following for the next ***three*** **(03)** items that follow :

Consider the binomial expansion of $(p + qx)^9$ :

**22.** What is the value of $q$ if the coefficients of $x^3$ and $x^6$ are equal ?

(a) $p$ (b) $9p$ (c) $1/p$ (d) $p^2$

**23.** What is the ratio of the coefficients of middle terms in the expansion (when expanded in ascending powers of $x$) ?

(a) $pq$ (b) $p/q$

(c) $4p/5q$ (d) $1/(pq)$

**24.** Under what condition the coefficients of $x^2$ and $x^4$ are equal ?

(a) $p : q = 7 : 2$ (b) $p^2 : q^2 = 7 : 2$

(c) $p : q = 2 : 7$ (b) $p^2 : q^2 = 2 : 7$

Consider the following for the next ***three*** **(03)** items that follow :

Consider the word 'QUESTION' :

**25.** How many 4-letter words each of two vowels and two consonants with or without meaning, can be formed ?

(a) 36 (b) 144 (c) 576 (d) 864

**26.** How many 8-letter words with or without meaning, can be formed such that consonants and vowels occupy alternate positions ?

(a) 288 (b) 576 (c) 1152 (d) 2304

**27.** How many 8-letter words with or without meaning, can be formed so that all consonants are together ?

(a) 5760 (b) 2880 (c) 1440 (d) 720

Consider the following for the next ***three*** **(03)** items that follow :

Let $\Delta$ be the determinant of a matrix $A$,

where $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ and $C_{11}$, $C_{12}$, $C_{13}$ be the cofactors of $a_{11}$, $a_{12}$, $a_{13}$ respectively.

**28.** What is the value of $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$ ?

(a) 0 (b) 1 (c) $\Delta$ (d) $-\Delta$

**29.** What is the value of $a_{21}\,C_{11} + a_{22}C_{12} + a_{23}C_{13}$?

(a) 0 (b) 1 (c) $\Delta$ (d) $-\Delta$

**30.** What is the value of $\begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{23} & a_{33} & a_{13} \\ a_{22} & a_{32} & a_{12} \end{vmatrix}$?

(a) 0 (b) 1 (c) $\Delta$ (d) $-\Delta$

Consider the following for the next ***three*** **(03)** items that follow :

Let $f(x)$ be a function satisfying $f(x + y) = f(x)f(y)$ for all $x,y \in N$ such that $f(1) = 2$:

**31.** If $\sum_{x=2}^{n} f(x)=2044$, then what is the value of $n$?

(a) 8 (b) 9 (c) 10 (d) 11

**32.** What is $\sum_{x=1}^{5} f(2x-1)$ equal to ?

(a) 341 (b) 682 (c) 1023 (d) 1364

**33.** What is $\sum_{x=1}^{6} 2^x f(x)$ equal to?

(a) 1365 (b) 2730 (c) 4024 (d) 5460

Consider the following for the next ***three*** **(03)** items that follow :

A university awarded medals in basket ball, football and volleyball. Only $x$ students ($x < 6$) got medal in all the three sports and the medals went to a total of $15x$ students. It awarded $5x$ medals in basketball, $(4x + 15)$ medals in football and $(x + 25)$ medals in volleyball.

**34.** How many received medals in exactly two of the three sports ?

(a) $30 - 4x$ (b) $35 - 7x$ (c) $40 - 7x$ (d) $45 - 5x$

**35.** How many received medals in at least two of three sports ?

(a) $30 - 6x$ (b) $35 - 6x$ (c) $40 - 5x$ (d) $40 - 6x$

**36.** How many received medals in exactly one of three sports ?

(a) $21x - 40$ (b) $21x - 35$

(c) $20x - 35$ (d) $20x - 25$

Consider the following for the next ***three*** **(03)** items that follow :

Let $A=\begin{pmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{pmatrix}$ and $A = P + Q$ where

$P$ is symmetric matrix and $Q$ is skew-symmetric matrix.

**37.** What is $P$ equal to?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $\cos 2\theta\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $\cos 2\theta\begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

**38.** What is $Q$ equal to ?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $\cos 2\theta\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $\cos 2\theta\begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

**39.** What is the minimum value of determinant of $A$ ?

(a) 1/4 (b) 1/2 (c) 3/4 (d) 1

Consider the following for the next ***three*** **(03)** items that follow :

$ABC$ is a triangular plot with $AB = 16$ $m$, $BC$= 10 m and $CA$ = 10 m. A lamp post is situated at the middle point of the side $AB$. The lamp post subtends an angle 45° at the vertex $B$.

**40.** What is the height of the lamp post ?

(a) 6 m (b) 7 m (c) 8 m (d) 9 m

**41.** What is $\dfrac{AB}{\sin C}$ equal to?

(a) 17 m (b) $\dfrac{50}{3}$ m (c) $\dfrac{40}{3}$ m (d) 16 m

**42.** What is $\cos A + \cos B + \cos C$ equal to ?

(a) 1 (b) $\dfrac{41}{25}$ (c) $\dfrac{37}{25}$ (d) $\dfrac{33}{25}$

Consider the following for the next ***three*** **(03)** items that follow :

There are two points $P$ and $Q$ due south of a leaning tower, which leans towards north. $P$ is at a distance $x$ and $Q$ is at a distance $y$ from the foot of the tower ($x > y$). The angles of elevation of the top of the tower from $P$ and $Q$ are 15° and 75° respectively.

**43.** At what height is the top of the tower above the ground level ?

(a) $\dfrac{x-y}{2\sqrt{3}}$ (b) $\dfrac{x-y}{4\sqrt{3}}$ (c) $\dfrac{x-y}{4}$ (d) $\dfrac{x-y}{2}$

**44.** If $\theta$ is the inclination of the tower to the horizontal, then what is $\cot\theta$ equal to?

(a) $2+\dfrac{\sqrt{3}(x-y)}{x+y}$ (b) $2-\dfrac{\sqrt{3}(x-y)}{x+y}$

(c) $2+\dfrac{\sqrt{3}(x+y)}{x-y}$ (d) $2-\dfrac{\sqrt{3}(x+y)}{x-y}$

**45.** What is the length of the tower ?

(a) $\dfrac{x-y}{2\sqrt{3}}\sqrt{1+\left\{2+\dfrac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(b) $\dfrac{x-y}{2\sqrt{3}}\sqrt{1+\left\{2-\dfrac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(c) $\dfrac{x-y}{4\sqrt{3}}\sqrt{1+\left\{2+\dfrac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(d) $\dfrac{x-y}{4\sqrt{3}}\sqrt{1+\left\{2-\dfrac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

**46.** What is the value of $\operatorname{cosec}\left(-\dfrac{73\pi}{3}\right)$?

(a) $\dfrac{2}{\sqrt{3}}$ (b) $-\dfrac{2}{\sqrt{3}}$ (c) 2 (d) –2

**47.** What is the value of

$\cos\left(\dfrac{5\pi}{17}\right)+\cos\left(\dfrac{7\pi}{17}\right)+2\cos\left(\dfrac{11\pi}{17}\right)\cos\left(\dfrac{\pi}{17}\right)$?

(a) 0 (b) 1

(c) $4\cos\left(\dfrac{6\pi}{17}\right)\cos\left(\dfrac{\pi}{17}\right)$ (d) $4\cos\left(\dfrac{11\pi}{17}\right)\cos\left(\dfrac{\pi}{17}\right)$

**48.** What is the value of $\tan\left(\dfrac{3\pi}{8}\right)$?

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$

(c) $1-\sqrt{2}$ (d) $-(\sqrt{2}+1)$

**49.** What is $\tan^{-1}\cot(\operatorname{cosec}^{-1}2)$ equal to?

(a) $\pi/8$ (b) $\pi/5$ (c) $\pi/4$ (d) $\pi/3$

**50.** In a triangle $ABC$, $a = 4$, $b = 3$, $c = 2$. What is $\cos 3C$ equal to ?

(a) 7/125 (b) 11/128 (c) 7/64 (d) 11/64

**51.** What is cos36° – cos72° equal to?

(a) $\dfrac{\sqrt{5}}{2}$ (b) $-\dfrac{\sqrt{5}}{2}$ (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

**52.** If $\sec x = \dfrac{25}{24}$ and $x$ lies in the fourth quadrant, then what is the value of $\tan x + \sin x$ ?

(a) $-\dfrac{625}{168}$ (b) $-\dfrac{343}{600}$ (c) $\dfrac{625}{168}$ (d) $\dfrac{343}{600}$

**53.** What is the value of $\tan^2 165° + \cot^2 165°$ ?

(a) 7 (b) 14 (c) $4\sqrt{3}$ (d) $8\sqrt{3}$

**54.** What is the value of $\sin\left(2n\pi+\dfrac{5\pi}{6}\right)\sin\left(2n\pi-\dfrac{5\pi}{6}\right)$, where $n \in Z$?

(a) –1/4 (b) –3/4 (c) 1/4 (d) 3/4

**55.** If $1 + 2(\sin x + \cos x)(\sin x - \cos x) = 0$ where $0 < x < 360°$, then how many values does $x$ take ?

(a) Only one value (b) Only two values

(c) Only three values (d) Four values

**56.** Consider the following statements in respect of the line passing through origin and inclining at an angle of 75° with the positive direction of $x$-axis :

1. The line passes through the point $\left(1, \dfrac{1}{2-\sqrt{3}}\right)$.
2. The line entirely lies in first and third quadrants.

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

**57.** If $P(3, 4)$ is the mid-point of a line segment between the axes, then what is the equation of the line ?

(a) $3x + 4y - 25 = 0$ (b) $4x + 3y - 24 = 0$

(c) $4x - 3y = 0$ (d) $3x - 4y + 7 = 0$

**58.** The base $AB$ of an equilateral triangle $ABC$ with side 8 cm lies along the $y$-axis such that the mid-point of $AB$ is at the origin and $B$ lies above the origin. What is the equation of line passing through (8, 0) and parallel to the side $AC$ ?

(a) $x-\sqrt{3}y-8=0$ (b) $x+\sqrt{3}y-8=0$

(c) $\sqrt{3}x+y-8\sqrt{3}=0$ (d) $\sqrt{3}x-y-8\sqrt{3}=0$

**59.** The centre of the circle passing through origin and making positive intercepts 4 and 6 on the coordinate axes, lies on the line

(a) $2x - y + 1 = 0$ (b) $3x - 2y - 1 = 0$

(c) $3x - 4y + 6 = 0$ (d) $2x + 3y - 26 = 0$

**60.** The centre of an ellipse is at (0, 0), major axis is on the $y$-axis. If the ellipse passes through (3, 2) and (1, 6), then what is its eccentricity ?

(a) $\sqrt{3}/2$ (b) $\sqrt{3}$ (c) $\sqrt{5}/2$ (d) $\sqrt{5}$

**61.** An equilateral triangle is inscribed in a parabola $x^2=\sqrt{3}y$ where one vertex of the triangle is at the vertex of the parabola. If $p$ is the length of side of the triangle and $q$ is the length of the latus rectum, then which one of the following is correct ?

(a) $p = q$ (b) $p=\sqrt{3}q$

(c) $p=2\sqrt{3}q$ (d) $2\sqrt{3}p=q$

**62.** Consider the points $A(2,4,6), B(-2,-4,-2), C(4,6,4)$ and $D(8, 14, 12)$. Which of the following statements is/are correct?

1. The points are the vertices of a rectangle $ABCD$.
2. The mid-point of $AC$ is the same as that of BD.

Select the correct answer using the code given below :

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

**63.** Consider the equation of a sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z - 16 = 0$. Which of the following statements is/are correct?

1. $z$-axis is tangent to the sphere.
2. The centre of the sphere lies on the plane $x + y + z - 9 = 0$.

Select the correct answer using the code given below :

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

**64.** A plane cuts intercepts 2, 2, 1 on the coordinate axes. What are the direction cosines of the normal to the plane ?

(a) <2/3, 2/3, 1/3> (b) <1/3, 2/3, 2/3>

(c) $<\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}>$ (d) $<\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}>$

**65.** Consider the following statements :

1. The direction ratios of $y$-axis can be <0, 4, 0>
2. The direction ratios of a line perpendicular to $z$-axis can be <5, 6, 0>

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

**66.** $PQRS$ is a parallelogram. If $\overrightarrow{PR}=\vec{a}$ and $\overrightarrow{QS}=\vec{b}$, then what is $\overrightarrow{PQ}$ equal to?

(a) $\vec{a}+\vec{b}$ (b) $\vec{a}-\vec{b}$ (c) $\frac{\vec{a}+\vec{b}}{2}$ (d) $\frac{\vec{a}-\vec{b}}{2}$

**67.** Let $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular. What is the angle between $\vec{a}$ and $\vec{b}$ ?

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

**68.** Let $\vec{a},\vec{b}$ and $\vec{c}$ be unit vectors lying on the same plane. What is $\left\{\left(3\vec{a}+2\vec{b}\right)\times\left(5\vec{a}-4\vec{c}\right)\right\}\cdot\left(\vec{b}+2\vec{c}\right)$ equal to?

(a) −8 (b) −32 (c) 8 (d) 0

**69.** What are the values of $x$ for which the angle between the vectors $2x^2\hat{i}+3x\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}+x^2\hat{k}$ is obtuse ?

(a) $0 < x < 2$ (b) $x < 0$

(c) $x > 2$ (d) $0 \le x \le 2$

**70.** The position vectors of vertices $A$, $B$ and $C$ of triangle $ABC$ are respectively $\hat{j}+\hat{k}, 3\hat{i}+\hat{j}+5\hat{k}$ and $3\hat{j}+3\hat{k}$. What is angle $C$ equal to ?

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

**71.** Let $z = [y]$ and $y = [x]-x$, where [.] is the greatest integer function. If $x$ is not an integer but positive, then what is the value of $z$ ?

(a) −1 (b) 0 (c) 1 (d) 2

**72.** If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $fog(x) = gof(x)$, then what is the value of $k$?

(a) 7 (b) 5 (c) 4 (d) 3

**73.** What is the minimum value of the function $f(x) = \log_{10}(x^2 + 2x + 11)$ ?

(a) 0 (b) 1 (c) 2 (d) 10

**74.** What is $\int(x^x)^2 (1 + \ln x)dx$ equal to?

(a) $x^{2x} + c$ (b) $\frac{1}{2}x^{2x}+c$

(c) $2x^{2x} + c$ (d) $\frac{1}{2}x^{x}+c$

**75.** What is $\int e^x \{1 + \ln x + x\ln x\}dx$ equal to?

(a) $xe^x\ln x + c$ (b) $x^2e^x\ln x + c$

(c) $x + e^x\ln x + c$ (d) $xe^x + \ln x + c$

**76.** What is $\int \frac{(\cos x)^{1.5}-(\sin x)^{1.5}}{\sqrt{\sin x\cdot\cos x}}dx$ equal to ?

(a) $\sqrt{\sin x}-\sqrt{\cos x}+c$ (b) $\sqrt{\sin x}+\sqrt{\cos x}+c$

(c) $2\sqrt{\sin x}+2\sqrt{\cos x}+c$

(d) $\frac{1}{2}\sqrt{\sin x}+\frac{1}{2}\sqrt{\cos x}+c$

**77.** If $y=\frac{x\sqrt{x^2-16}}{2}-8\,ln\left|x+\sqrt{x^2-16}\right|$, then what is $\frac{dy}{dx}$ equal to ?

(a) $x\sqrt{x^2-16}$ (b) $x-\sqrt{x^2-16}$

(c) $\sqrt{x^2-16}$ (d) $4\sqrt{x^2-16}$

**78.** If $y = (x^x)^x$, then which one of the following is correct ?

(a) $\frac{dy}{dx}+xy(1+2lnx)=0$ (b) $\frac{dy}{dx}-xy(1+2lnx)=0$

(c) $\frac{dy}{dx}-2xy(1+lnx)=0$ (d) $\frac{dy}{dx}+2xy(1+lnx)=0$

**79.** What is the maximum value of $3(\sin x - \cos x) + 4(\cos^3 x - \sin^3 x)$ ?

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

**80.** What is the area of the region (in the first quadrant) bounded by $y=\sqrt{1-x^2}$, $y=x$ and $y=0$ ?

(a) $\pi/4$ (b) $\pi/6$ (c) $\pi/8$ (d) $\pi/12$

**81.** What is the area of the region bounded by $x-|y|=0$ and $x-2=0$ ?

(a) 1 (b) 2 (c) 4 (d) 8

**82.** If $f(\alpha)=\sqrt{\sec^2\alpha-1}$, then what is $\frac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)}$ equal to ?

(a) $f(\alpha-\beta)$ (b) $f(\alpha+\beta)$

(c) $f(a)f(\beta)$ (d) $f(\alpha\beta)$

**83.** If $f(x)=\ln(x+\sqrt{1+x^2})$, then which one of the following is correct ?

(a) $f(x)+f(-x)=0$ (b) $f(x)-f(-x)=0$

(c) $2f(x)=f(-x)$ (d) $f(x)=2f(-x)$

**84.** What is $\lim_{x\to 0}\frac{x}{\sqrt{1-\cos 4x}}$ equal to ?

(a) $\frac{1}{2\sqrt{2}}$ (b) $-\frac{1}{2\sqrt{2}}$

(c) $\sqrt{2}$ (d) Limit does not exist

**85.** What is $\lim_{x\to\frac{\pi}{2}}\frac{4x-2\pi}{\cos x}$ equal to ?

(a) $-4$ (b) $-2$ (c) 2 (d) 4

**86.** If $f(x)=\frac{x^2+x+|x|}{x}$, then what is $\lim_{x\to 0}f(x)$ equal to ?

(a) 0 (b) 1 (c) 2

(d) $\lim_{x\to 0}f(x)$ does not exist

**87.** What is $\lim_{h\to 0}\frac{\sin^2(x+h)-\sin^2 x}{h}$ equal to ?

(a) $\sin^2 x$ (b) $\cos 2x$ (c) $\sin 2x$ (d) $\cos 2x$

**88.** Let $f(x)$ be a function such that $f'(x) = g(x)$ and $f''(x) = -f(x)$. Let $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. Then consider the following statements :

1. $h'(3) = 0$ 2. $h(l) = h(2)$

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both I and 2 (d) Neither 1 nor 2

**89.** If $y=ln^2\left(\frac{x^2-x+1}{x^2+x+1}\right)$, then what is $\frac{dy}{dx}$ at $x = 0$ equal to?

(a) $-2$ (b) 0 (c) 1 (d) 2

**90.** If $\frac{d}{dx}\left(\frac{1+x^4+x^8}{1-x^2+x^4}\right)=ax+bx^3$, then which one of the following is correct ?

(a) $a = b$ (b) $a = 2b$

(c) $a + b = 0$ (d) $2a = b$

**91.** Under which one of the following conditions does the function $f(x) = (p\sec x)^2 + (q\,\mathrm{cosec}\,x)^2$ attain minimum value ?

(a) $\tan^2 x = q/p$ (b) $\cot^2 x = q/p$

(c) $\tan^2 x = pq$ (d) $\cot^2 x = pq$

**92.** Where does the function $f(x)=\sum_{j=1}^{7}(x-j)^2$ attain its minimum value ?

(a) $x = 3.5$ (b) $x = 4$ (c) $x = 4.5$ (d) $x = 5$

**93.** Consider the following statements in respect of the function $f(x)=\begin{cases}|x|+1, & 0<|x|\le 3\\ 1, & x=0\end{cases}$

1. The function attains maximum value only at $x = 3$
2. The function attains local minimum only at $x = 0$

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**94.** What is $\int_0^1 ln\left(\frac{1}{x}-1\right)dx$
(a) –1 (b) 0 (c) 1 (d) *ln*2

**95.** If $\int_0^{\pi/2}(\sin^4 x+\cos^4 x)dx=k$, then what is the value of $\int_0^{20\pi}(\sin^4 x+\cos^4 x)dx$ ?
(a) $k$ (b) $10k$ (c) $20k$ (d) $40k$

**96.** What is $\int_{-\pi/2}^{\pi/2}(e^{\cos x}\sin x+e^{\sin x}\cos x)dx$ equal to?
(a) $\frac{e^2-1}{e}$ (b) $\frac{e^2+1}{e}$ (c) $\frac{1-e^2}{e}$ (d) 0

**97.** What is the area of the region enclosed in the first quadrant by $x^2+y^2=\pi^2$, $y=\sin x$ and $x=0$ ?
(a) $\frac{\pi^3}{4}-1$ (b) $\frac{\pi^3}{4}-2$ (c) $\frac{\pi^3}{2}-1$ (d) $\frac{\pi^2}{4}-2$

**98.** Consider the following statements :
1. The degree of the differential equation : $\frac{dy}{dx}+\cos\left(\frac{dy}{dx}\right)=0$ is 1.
2. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\cos\left(\frac{dy}{dx}\right)=0$ is 2.

Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**99.** What is the differential equation of the family of parabolas having vertex at origin and axis along positive $y$-axis ?
(a) $x\frac{dy}{dx}+2y=0$ (b) $x\frac{dy}{dx}-2y=0$
(c) $y\frac{dx}{dy}+2x=0$ (d) $y\frac{dx}{dy}-2x=0$

**100.** What is the solution of the differential equation $(dy-dx)+\cos x(dy+dx)=0$ ?
(a) $y=\tan\left(\frac{x}{2}\right)-x+c$ (b) $y=\frac{1}{2}\tan\left(\frac{x}{2}\right)-x+c$
(c) $y=2\tan\left(\frac{x}{2}\right)-x+c$ (d) $y=\tan\left(\frac{x}{2}\right)-2x+c$

**101.** Let $x$ be the mean of squares of first $n$ natural numbers and $y$ be the square of mean of first $n$ natural numbers. If $\frac{x}{y}=\frac{55}{42}$, then what is the value of $n$ ?
(a) 24 (b) 25 (c) 27 (d) 30

**102.** What is the probability of getting a composite number in the list of natural numbers from 1 to 50 ?
(a) 7/10 (b) 17/25 (c) 18/25 (d) 33/50

**103.** If $n>7$, then what is the probability that $C(n, 7)$ is a multiple of 7 ?
(a) 0 (b) 1/7 (c) 1/2 (d) 1

**104.** Two numbers $x$ and $y$ are chosen at random from a set of first 10 natural numbers. What is the probability that $(x+y)$ is divisible by 4 ?
(a) 1/5 (b) 2/9 (c) 8/45 (d) 7/45

**105.** A number $x$ is chosen at random from first $n$ natural numbers. What is the probability that the number chosen satisfies $x+\frac{1}{x}>2$?
(a) $1/n$ (b) $1/(2n)$ (c) $(n-1)/n$ (d) 1

**106.** Three fair dice are tossed once. What is the probability that they show different numbers that are in *AP* ?
(a) 1/12 (b) 1/18 (c) 1/36 (d) 1/72

**107.** If $P(A)=0.5$, $P(B)=0.7$ and $P(A\cap B)=0.3$, then what is the value of $P(A'\cap B')+P(A'\cap B)+P(A\cap B')$?
(a) 0.6 (b) 0.7 (c) 0.8 (d) 0.9

**108.** Five coins are tossed once. What is the probability of getting at most four tails?
(a) 31/32 (b) 15/16 (c) 29/32 (d) 7/8

**109.** Three fair dice are thrown. What is the probability of getting a total greater than or equal to 15 ?
(a) 19/216 (b) 1/12 (c) 17/216 (d) 5/54

**110.** The probability that a person hits a target is 0.5. What is the probability of at least one hit in 4 shots ?
(a) 1/8 (b) 1/16 (c) 15/16 (d) 7/8

**111.** A box contains 2 white balls, 3 black balls and 4 red balls. What is the number of ways of drawing 3 balls from the box with at least one black ball?
(a) 84 (b) 72 (c) 64 (d) 48

**112.** During war one ship out of 5 was sunk on an average in making a certain voyage. What is the probability that exactly 3 out of 5 ships would arrive safely ?
(a) 16/625 (b) 32/625 (c) 64/625 (d) 128/625

**113.** A card is drawn from a pack of 52 cards. A gambler bets that it is either a spade or an ace. The odds

against his winning are
(a) 9 : 4 (b) 35 : 17 (c) 17 : 35 (d) 4 : 9

**114.** The coefficient of correlation between ages of husband and wife at the time of marriage for a given set of 100 couples was noted to be 0.7. Assume that all these couples survive to celebrate the silver jubilee of their marriage. The coefficient of correlation at that point of time will be
(a) 1 (b) 0.9 (c) 0.7 (d) 0.3

**115.** The completion of a construction job may be delayed due to strike. The probability of strike is 0.6. The probability that the construction job gets completed on time if there is no strike is 0.85 and the probability that the construction job gets completed on time if there is a strike is 0.35. What is the probability that the construction job will not be completed on time ?
(a) 0.35 (b) 0.45 (c) 0.55 (d) 0.65

Consider the following for the next ***two*** **(02)** items that follow :

The mean and standard deviation (SD) of marks obtained by 50 students of a class in 4 subjects are given below :

| Subject | Mathematics | Physics | Chemistry | Biology |
|---|---|---|---|---|
| Mean Marks | 40 | 28 | 38 | 36 |
| SD | 15 | 12 | 14 | 16 |

**116.** Which one of the following subjects shows highest variability of marks ?
(a) Mathematics (b) Physics
(c) Chemistry (d) Biology

**117.** What is the coefficient of variation of marks in Mathematics?
(a) 37.5% (b) 38.0% (c) 38.5% (d) 39.0%

Consider the following for the next ***three*** **(03)** items that follow :

Consider the following grouped frequency distribution :

| Class | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| Fre-quency | 1 | 2 | 4 | 6 | 4 | 3 |

**118.** What is the median of the distribution ?
(a) 34 (b) 34.5 (c) 35 (d) 35.5

**119.** What is mean deviation about the median ?
(a) 11.4 (b) 11.1 (c) 10.8 (d) 10.5

**120.** What is the mean deviation about the mean ?
(a) 10.15 (b) 10.65 (c) 11.15 (d) 11. 65

## SOLUTIONS

**1. (a) :** Number of four digit natural number in which all digits are odd $= 5 \times 5 \times 5 \times 5 = 625$

**2. (b) :** $\sum_{r=0}^{n} 2^r C(n, r) = \sum_{r=0}^{n} {}^nC_r\, 2^r = (1+2)^n = 3^n$

**3. (c) :** In the given word, MATHEMATICS
M – 2, A – 2, T – 2, H – 1, E – 1, I – 1, C – 1, S – 1
Total letters are 11.
Required number of words = 10! / 2!2!= 907200

**4. (b) :** Let $x = 2, y = 3$
$xy = 6$ and $x - y = -1$
If $f(xy) = x - y = f(2 \times 3) = 2 - 3 = -1$ ....(i)
let $x = 3, y = 2$
$xy = 6$ and $x - y = 1$
$f(6) = f(xy) = x - y = 1$ ....(ii)
From (i) and (ii), we get
$f(6)$ can take 2 values.
Thus, $f$ is not a function from $Z$ to $Z$.
Clearly, function given in statement (2) is from $N$ to $N$.

**5. (a) :** $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\Delta = a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} - a_{23}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} + a_{33}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$
$$= yz(z-y) - zx(z-x) + xy(y-x)$$
$$= x^2(z-y) - x(z^2-y^2) + yz(z-y)$$
$$= (z-y)[x^2 - x(z+y) + yz] = (z-y)(z-x)(y-x)$$

**6. (d) :** $|A| = -\cos^2\theta - \sin^2\theta = -1$

$$\text{adj}(A) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj}(A)}{|A|} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix} = A$$

1. $A + \text{adj}(A) =$ Null matrix
2. $A^{-1} + \text{adj}(A) = A^{-1} - A^{-1} =$ Null matrix
3. $A - A^{-1} =$ Null matrix

So, statements 1, 2 and 3 are correct.

**7. (d) :** Since, $X \to 3 \times 3$ matrix, $Y \to 2 \times 3$ matrix, and $Z \to 3 \times 2$ matrix $\therefore$ $ZY$ is $3 \times 3$ matrix
$(ZY)X$ is a $3 \times 3$ matrix and has 9 entries.
Statement 1 is true.
Now, $XZ$ is $3 \times 2$ matrix
$\therefore$ $Y(XZ)$ is a $2 \times 2$ matrix

The matrix $Y(XZ)$ have 4 entries

$\therefore$ Statement 2 is correct.

Now, $YZ$ is $2 \times 2$ matrix.

$X$ is $3 \times 3$ matrix.

$\therefore$ $X(YZ)$ is not defined.

$\therefore$ Statement 3 is correct.

Hence, statement 1, 2 and 3 are correct.

**8. (d) :** Sum of roots $= -\frac{b}{a}$ and Product of roots $= \frac{c}{a}$

Given that, sum of roots = product of roots

$-\frac{b}{a} = \frac{c}{a} \Rightarrow c = -b \quad \Rightarrow b + c = 0$

$\therefore$ Infinite number of quadratic equations are possible.

**9. (a)**

**10. (a) :** 1. $2 + 4 + 6 + 8 + \text{---} \; 2n$

$= \frac{n}{2}[2 + 2n] \quad = n(n+1) \; = \; n^2 + n$, which is correct.

2. $P(n) = n^2 + n + 41$

$P(1) = 1 + 1 + 41 = 43$, which is a prime number.

$P(2) = 4 + 2 + 41 = 47$, which is a prime number.

But $P(41) = 41^2 + 41 + 41 = 1763$, which is divisible by 41, so it is not a prime number, which is wrong.

**11. (a) :** $p$, $q$ be the roots of the quadratic equation $x^2 + bx + c = 0$. Sum of roots, $p + q = -b$

Product of roots, $pq = c$

$p^2 + q^2 = 11\,pq$ [Given]

$(p - q)^2 = 9\,pq$

$\Rightarrow (p - q)^2 = 9 \times c \;\; \Rightarrow p - q = \pm 3\sqrt{c}$

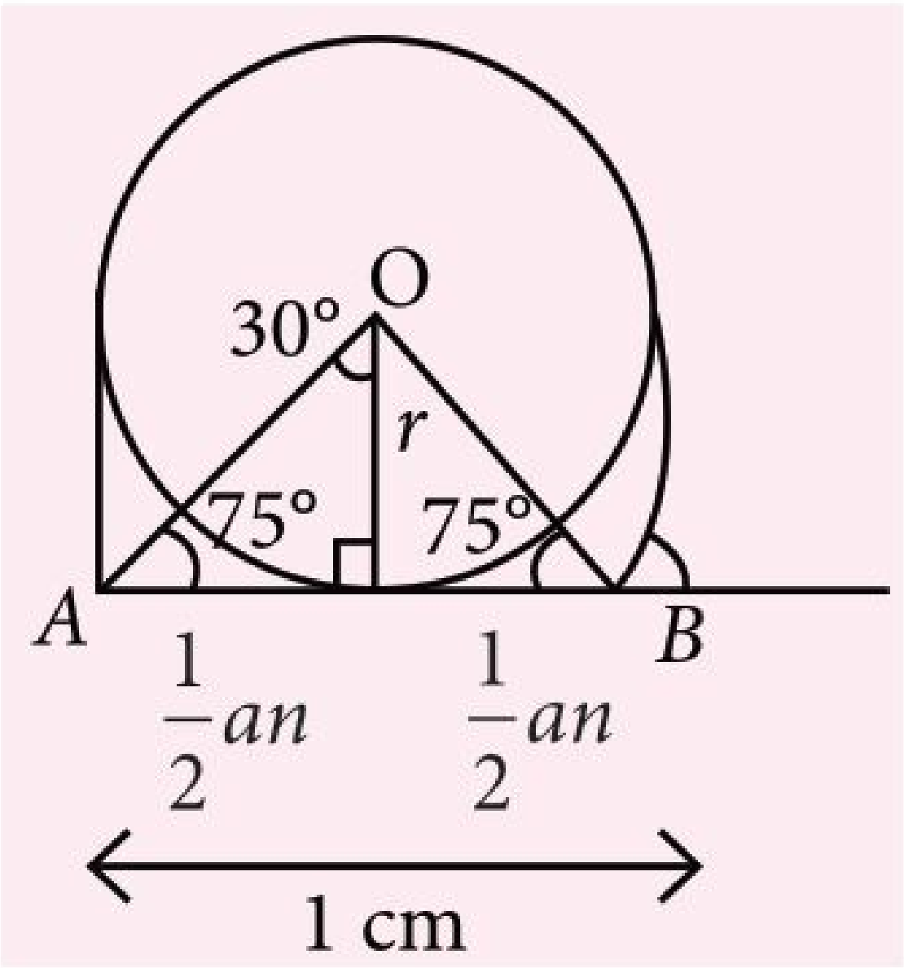
$\therefore p - q = 3\sqrt{c}$ $\quad [-3\sqrt{c}$ is not possible $\because p > q]$

**12. (c) :** Exterior angle of polygon $= \frac{360^\circ}{n} = \frac{360^\circ}{12} = 30^\circ$

So, Interior angle of polygon = 180° – 30° = 150°

$OA$ bisets the interior angle, So,

$\angle\, OAD = 75^\circ$



In $\Delta\, OAD$, $\tan 75^\circ = \frac{OD}{AD}$

$\Rightarrow \tan(30^\circ + 45^\circ) = \frac{r}{1/2}$

$$\Rightarrow \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = 2r$$

$$\Rightarrow \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} = 2r \quad \Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2r \Rightarrow \frac{4 + 2\sqrt{3}}{2} = 2r$$

$\Rightarrow \quad d = 2 + \sqrt{3}$

**13. (c) :** H(7) = 7 is the highest prime factor

Similarly, H(8) = 2; H(9) = 3; H(10) = 5; H(11) = 11; H(12) = 3; H(13) = 13; H(14) = 7; H(15) = 5; H(16) = 2;

Total distinct highest prime factors are 2, 3, 5, 7, 11, 13 *i.e.*, 6 in number.

**14. (d) :** $R : \{(x, y) : x, y \in N \text{ and } x^2 = y^3\}$

Let $x = 2$, $y = 3$

1. For $(x, x) \in R \; \forall \; x \in N$

$2^2 \neq 2^3 \quad \Rightarrow \quad 4 \neq 8$

This is not correct statement.

2. $(x, y) \in R \Rightarrow \; (y, x) \in R$

$4 \neq 27$ and $9 \neq 8$

This statement is also not correct.

3. $(x, y) \in R$ and $(y, z) \in R$

$x^2 = y^3$ and $y^2 = z^3$

but $x^2 \neq z^3$

This statement is also not correct.

**15. (d) :** let $A = \{1, 3, 5, 7\}$; $B = \{3, 5\}$ and $C = \{3, 5, 8\}$

$A \cap B = \{3, 5\} = A \cap C$

But $B \neq C$

Now, let $A = \{1, 2, 3, 4\}$, $B = \{3, 4\}$ and $C = \{1, 2, 3\}$

$A \cup B = \{1, 2, 3, 4\} = A \cup C$,

which is equal to $A$. But $B \neq C$

This statement is not correct.

**16. (a) :** $|z| = \left|\frac{1 + i\sin\theta}{1 - i\sin\theta}\right| = \frac{\sqrt{1 + \sin^2\theta}}{\sqrt{1 + \sin^2\theta}} = 1$

**17. (c) :** $z = \frac{1 + i\sin\theta}{1 - i\sin\theta}$

$$\Rightarrow x + iy = \frac{1 + i\sin\theta}{1 - i\sin\theta} \times \frac{1 + i\sin\theta}{1 + i\sin\theta} = \frac{1 - \sin^2\theta + 2i\sin\theta}{1 + \sin^2\theta}$$

$$= \frac{1 - \sin^2\theta}{1 + \sin^2\theta} + \frac{2i\sin\theta}{1 + \sin^2\theta}$$

For purely real $z$,

$\frac{2\sin\theta}{1 + \sin^2\theta} = 0 \;\; \Rightarrow \;\; \sin\theta = 0 \;\; \Rightarrow \theta = n\pi$

**18. (b) :** We have $Z = \frac{1 - \sin^2\theta}{1 + \sin^2\theta} + \frac{2i\sin\theta}{1 + \sin^2\theta}$

For purely imaginary $z$,

$\frac{1 - \sin^2\theta}{1 + \sin^2\theta} = 0 \Rightarrow \sin^2\theta = 1 \;\; \Rightarrow \theta = (2n + 1)\frac{\pi}{2}$

**19. (d) :** $\frac{P}{Q} = \frac{5n + 4}{9n + 6}$

$$\Rightarrow \frac{\text{First term of A}}{\text{First term of B}} = \frac{5(1) + 4}{9(1) + 6} = \frac{3}{5} \Rightarrow \frac{A_1}{B_1} = \frac{9}{15} = \frac{3}{5}$$

**20. (c) :** $\frac{P_n}{Q_n} = \frac{5n + 4}{9n + 6} \quad \Rightarrow \frac{P_n}{Q_n} = \frac{\frac{n}{2}[2 \cdot 9 + 10\,(n-1)]}{\frac{n}{2}[2 \cdot 15 + 18\,(n-1)]}$

$\therefore \frac{A_{10}}{B_{10}} = \frac{9 + 90}{15 + 162} = \frac{99}{177} = \frac{33}{59}$

**21. (a)**

**22. (a) :** Given binomial expansion is $(p + qx)^9$.
$T_4 = {}^9C_3\, p^3 q^6 x^6$ ; $T_7 = {}^9C_6\, p^6 q^3 x^3$
Coefficient of $x^3$ = Coefficient of $x^6$
$\Rightarrow\ {}^9C_6\, p^6 q^3 = {}^9C_3\, p^3 q^6 \Rightarrow p^3 = q^3 \Rightarrow p = q$

**23. (b) :** Here, $n$ is odd, so it has middle terms

$T_{\left(\frac{9+1}{2}\right)}$ and $T_{\left(\frac{9+3}{2}\right)}$

$T_5 = {}^9C_4\,(qx)^4\,p^5$ and $T_6 = {}^9C_5\,(qx)^5\,p^4$

$$\therefore \frac{T_5}{T_6} = \frac{{}^9C_4\, q^4 p^5}{{}^9C_5\, q^5 p^4} \Rightarrow = \frac{p}{q}$$

**24. (b) :** $T_3 = {}^9\text{C}_2\, p^7 (qx)^2$; $T_5 = {}^9\text{C}_4\, p^5\, (qx)^4$
Coefficient of $x^2$ = Coefficient of $x^4 \Rightarrow {}^9C_2 q^2 p^7 = {}^9C_4 q^4 p^5$

$$\Rightarrow \frac{9!}{2!7!}p^2 = \frac{9!}{4!5!}q^2 \quad \Rightarrow \frac{p^2}{q^2} = \frac{7}{2}$$

**25. (d) :** In the given word 'QUESTION',
Vowels $\rightarrow 4$, Consonants $\rightarrow 4$
Number of 4 letters word with or without meaning
$= {}^4C_2 \times {}^4C_2 \times 4! = 36 \times 24 = 864$

**26. (c) :** Number of 8 letters word with or without meaning can be formed with vowels and consonants are at alternate position $= 4! \times 4! \times 2! = 576 \times 2 = 1152$

**27. (b) :** Number of 8 letters words with or without meaning can be formed such that consonants are together $= 5! \times 4! = 2880$

**28. (c) :** $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\text{C}_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix};\ \text{C}_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \text{ and } \text{C}_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$\therefore a_{11}\,\text{C}_{11} + a_{12}\,\text{C}_{12} + a_{13}\,\text{C}_{13}$

$$= a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \Delta$$

**29. (a) :** $a_{21}\,\text{C}_{11} + a_{22}\,\text{C}_{12} + a_{23}\,\text{C}_{13}$
$= a_{21}\,[a_{22} \cdot a_{33} - a_{23} \cdot a_{32}] - a_{22}\,[a_{21} \cdot a_{33} - a_{23} \cdot a_{31}]$
$+ a_{23}\,[a_{21} \cdot a_{32} - a_{22} \cdot a_{31}]$
$= a_{21}\, a_{22}\, a_{33} - a_{21} \cdot a_{23}\, a_{32} - a_{21} \cdot a_{22} \cdot a_{33} + a_{22} \cdot a_{23} \cdot a_{31}$
$+ a_{23} \cdot a_{21} \cdot a_{32} - a_{23} \cdot a_{22} \cdot a_{31}$
$= 0$

**30. (d)**

**31. (c) :** $f(x + y) = f(x) \cdot f(y)$
put $x = 1, y = 1$
$f(2) = f(1) \cdot f(1) \Rightarrow f(2) = 4$
Now, put $x = 2, y = 1$
$\Rightarrow f(3) = f(2) \cdot f(1) = 2^2 \cdot 2 = 8$
$\Rightarrow f(4) = f(3) \cdot f(1) = 8 \cdot 2 = 16$

$$\text{Now, } \sum_{x=2}^{n} f(x) = 2044$$

$\Rightarrow f(2) + f(3) + f(4) + --- + f(n) = 2044$
$\Rightarrow 4 + 8 + 16 + --- + 2^n = 2044$
$\Rightarrow 4[1 + 2^1 + 2^2 + --- + 2^{n-2}] = 2044$

$$\Rightarrow \frac{2^{n-1}-1}{2-1} = \frac{2044}{4} \Rightarrow 2^{n-1} = 512 \Rightarrow 2^{n-1} = 2^9 \Rightarrow n = 10$$

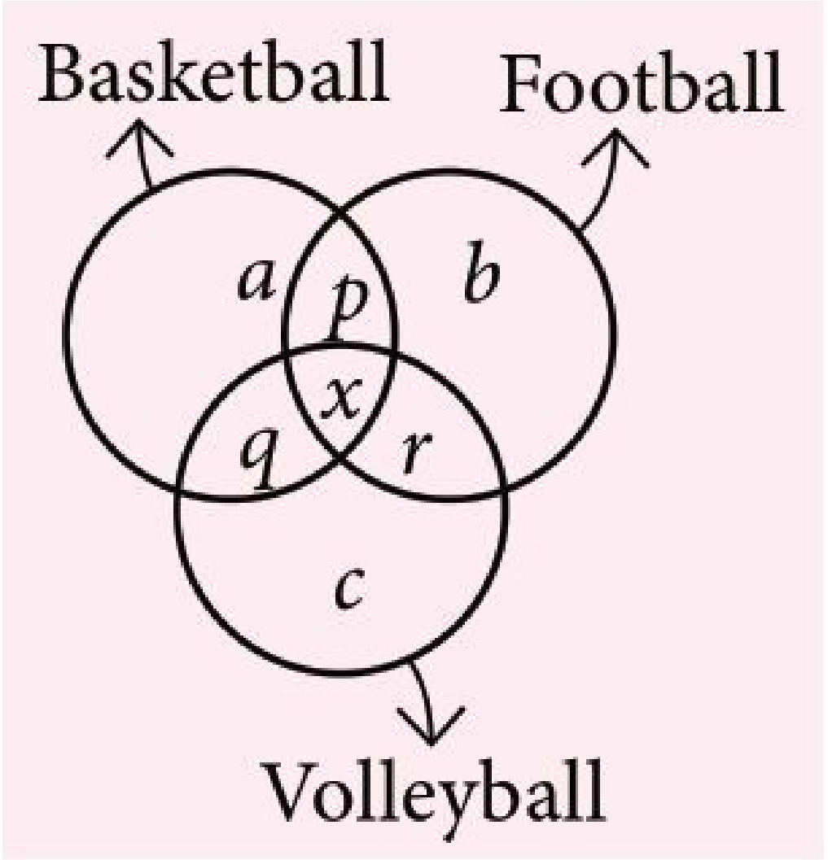
**32. (b) :** $\sum_{x=1}^{5} f(2x-1) = f(1) + f(3) + f(5) + f(7) + f(9)$

$= 2 + 2^3 + 2^5 + 2^7 + 2^9 = 682$

**33. (d) :** $\sum_{x=1}^{6} 2^x f(x)$

$= 2\,f(1) + 2^2 f(2) + 2^3 f(3) + 2^4 f(4) + 2^5 f(5) + 2^6 f(6)$
$= 2 \cdot 2 + 2^2 \cdot 2^2 + 2^3 \cdot 2^3 + 2^4 \cdot 2^4 + 2^5 \cdot 2^5 + 2^6 \cdot 2^6$
$= 2^2 + 2^4 + 2^6 + 2^8 + 2^{10} + 2^{12} = 5460$

**34. (c) :**



$a + b + c + p + q + r + x = 15\,x$ ...(i)
$\Rightarrow a + b + c + p + q + r = 14\,x$ ...(ii)
Now, $a + p + q + x = 5x$ ...(iii)
$b + p + r + x = 4x + 15$ ...(iv)
$c + q + r + x = x + 25$ ...(v)
From (iii) $a + p + q = 4x$ ...(vi)
$b + p + r = 3x + 15$ ...(vii) [from (iv)]
$c + q + r = 25$ ...(viii) [from (v)]
Adding (vi), (vii) and (viii), we get
$a + b + c + 2p + 2q + 2r = 7x + 40$
$\Rightarrow p + q + r = 7x + 40 - a - b - c - p - q - r$
$\Rightarrow p + q + r = 7x + 40 - 14\,x \Rightarrow p + q + r = 40 - 7x$

**35. (d) :** Medals received in at least two of three sports
$= p + q + r + x = 40 - 7x + x = 40 - 6x$

**36. (a) :** Medals received in exactly one of three sports
$= a + b + c = 14\,x - p - q - r$ [from (ii)]
$= 14\,x - [40 - 7x] = 21x - 40$

**37. (a) :** $A = P + Q$

$\because$ $P$ is symmetric $\Rightarrow P = \frac{1}{2}(A + A^T)$

$$= \frac{1}{2}\begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & \cos^2\theta & \sin^2\theta \\ \sin^2\theta & 0 & \cos^2\theta \\ \cos^2\theta & \sin^2\theta & 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

**38. (d) :** $\because$ $Q$ is skew symmetric $\Rightarrow Q = \frac{1}{2}[A - A^T]$

$$= \frac{1}{2}\left[\begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix} - \begin{bmatrix} 0 & \cos^2\theta & \sin^2\theta \\ \sin^2\theta & 0 & \cos^2\theta \\ \cos^2\theta & \sin^2\theta & 0 \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 0 & \sin^2\theta - \cos^2\theta & \cos^2\theta - \sin^2\theta \\ \cos^2\theta - \sin^2\theta & 0 & \sin^2\theta - \cos^2\theta \\ \sin^2\theta - \cos^2\theta & \cos^2\theta - \sin^2\theta & 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -\cos 2\theta & \cos 2\theta \\ \cos 2\theta & 0 & -\cos 2\theta \\ -\cos 2\theta & \cos 2\theta & 0 \end{bmatrix}$$

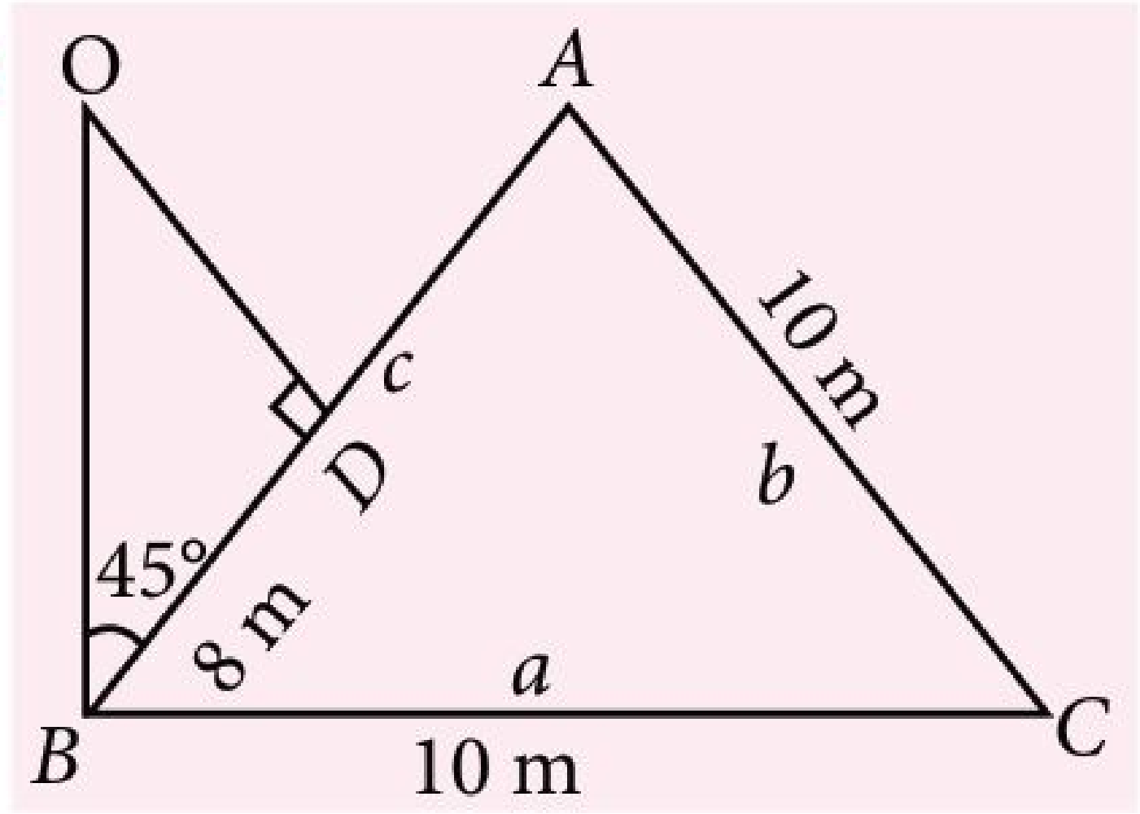
$$= \cos 2\theta \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

**39. (a) :** $|A| = 0 - \sin^2\theta\,(0 - \sin^4\theta) + \cos^2\theta(\cos^4\theta - 0)$
$= \sin^6\theta + \cos^6\theta$

For minimum value, put $\theta = 45°$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6 = \frac{1}{4}$$

**40. (c) :**



let, $OD$ be the height of lamp post in $\Delta OBD$

$$\tan 45° = \frac{OD}{BD} \Rightarrow 1 = \frac{OD}{8} \Rightarrow OD = 8 \text{ m}$$

**41. (b) :** $s = \frac{a+b+c}{2} = \frac{16+10+10}{2} = \frac{36}{2} = 18\text{m}$

Area of triangle $\Delta = \sqrt{18 \times 8 \times 2 \times 8} = \sqrt{144 \times 16}$
$= 12 \times 4 = 48 \text{ m}^2$

$$R = \frac{abc}{4\Delta} = \frac{10 \times 10 \times 16}{4 \times 48} = \frac{25}{3}\,;\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

## Students know 4-5 yrs at IIT-B will take them to next level: Director

At the close of round one of admissions on September 23, a look at options exercised by the top 1,000 JEE candidates reveals that Bombay, Delhi, Roorkee and Madras are top choices though Kanpur, Kharagpur and Hyderabad too have a sizeable representation.

Explaining the reason for most of the top 100 candidates opting for IIT Bombay, its director Subhasis Chaudhuri said: "IIT-B meets the aspirations of India's talented youth and challenges them to bring out their best. From giving them all the facilities to providing the right guidance, from accessing top-class education to getting the right platform, students know their investment of four or five years at IIT-B will take them to next level"

A total of 2.14 lakh registered for seat allocation through the Joint Seat Allocation Authority this year. Of these, 1.96 candidates - 1.51 lakh males, 45,370 females and three from the third gender - have filled in choices for seats in participating institutions.

The total number of seats available this year are 54,477 in 114 institutions, including IITs, NITs, IIEST Shibpur, IIITs and other government-funded technical institutes across the country. Going by choices, IIT Kharagpur has the maximum seats and variety of courses, said Prof Suryanarayana Doolla, organising chairman, JEE (Advanced). Analytics of applications received showed the lion's share of candidates opted for computer science, electrical engineering, electronics and communication engineering and mechanical engineering. About 30,000 students opted for biotech/biomedical engineering in line with the trend of the previous year.

*Courtesy : The Times of India*

$$\Rightarrow \frac{AB}{\sin C} = \frac{c}{\sin C} = 2R \;\Rightarrow \frac{AB}{\sin C} = 2\times\frac{25}{3} = \frac{50}{3}\text{ m}$$

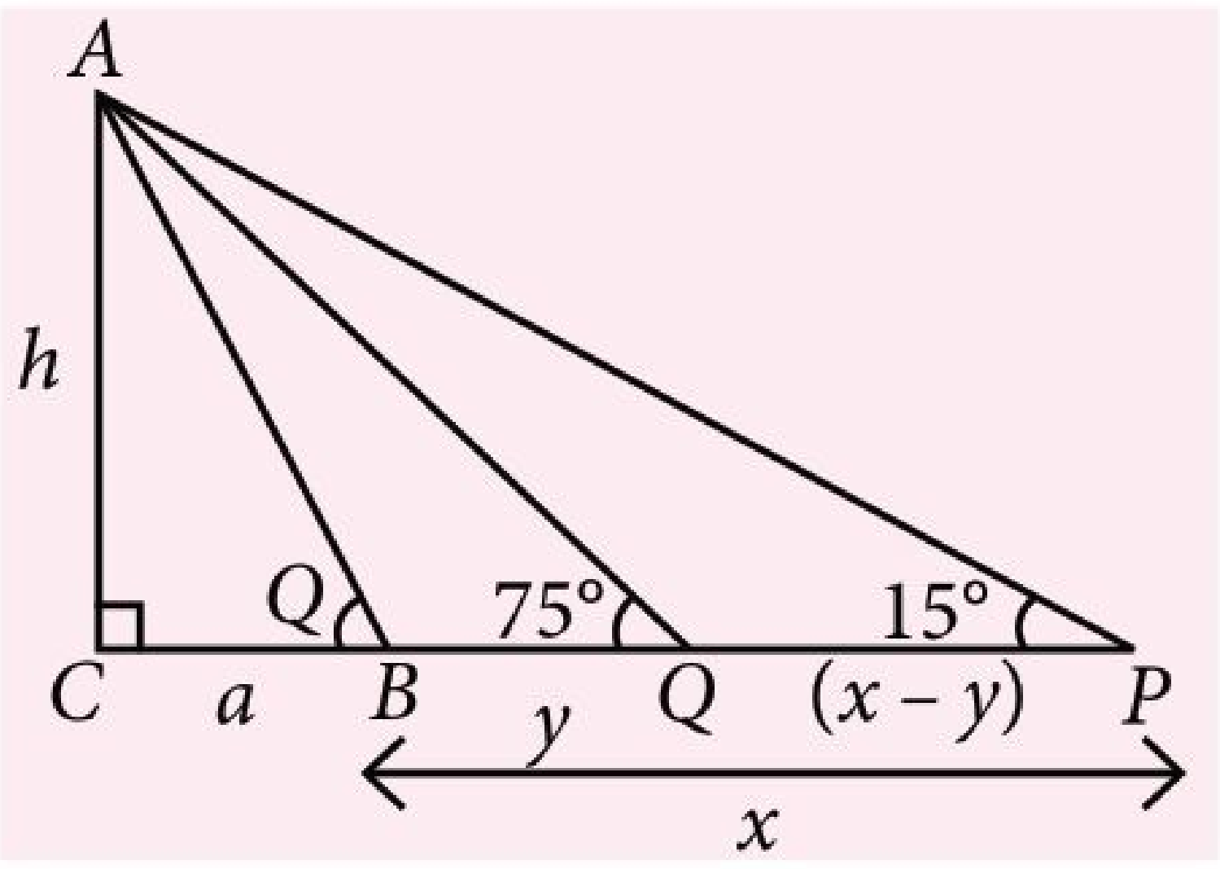
**42. (d) :** $\cos A + \cos B + \cos C$

$$= \frac{b^2+c^2-a^2}{2bc} + \frac{c^2+a^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{10^2+16^2-10^2}{2\times10\times16} + \frac{16^2+10^2-10^2}{2\times10\times16} + \frac{10^2+10^2-16^2}{2\times10\times10}$$

$$= \frac{4}{5}+\frac{4}{5}-\frac{56}{200} \;= \frac{8}{5}-\frac{7}{25} = \frac{33}{25}$$

**43. (a) :**



let $BC = a$

$AB$ is leaning tower and height $AC = h$

In $\Delta ACQ$, $\tan 75^\circ = \dfrac{h}{a+y} \Rightarrow a + y = h\cot 75^\circ$ ...(i)

In $\Delta ACP$, $\tan 15^\circ = \dfrac{h}{a+x}$

$\Rightarrow a + x = h\cot 15^\circ$ ...(ii)

Subtracting (ii) from (i), we get,

$a + x - a - y = h\cot 15^\circ - h\cot 75^\circ$

$$\Rightarrow x - y = h\tan 75^\circ - \frac{h}{\tan 75^\circ}$$

$$\Rightarrow x - y = h(2+\sqrt{3}) - \frac{h}{2+\sqrt{3}}$$

$$\Rightarrow x - y = h(2+\sqrt{3}) - h(2-\sqrt{3}) \Rightarrow h = \frac{x-y}{2\sqrt{3}}$$

**44. (d) :** $a + x = h\cot 15^\circ$

$$\Rightarrow a = \frac{x-y}{2\sqrt{3}}\times\tan 75^\circ - x = \frac{x-y}{2\sqrt{3}}\times(\sqrt{3}+2) - x$$

$$= \frac{(x-y)(2+\sqrt{3}) - 2\sqrt{3}x}{2\sqrt{3}}$$

In $\Delta ABC$, $\cot\theta = \dfrac{a}{h} = \dfrac{[(x-y)(2+\sqrt{3})-2\sqrt{3}x]}{2\sqrt{3}\times(x-y)}\times 2\sqrt{3}$

$$= 2 + \frac{\sqrt{3}x - \sqrt{3}y - 2\sqrt{3}x}{x-y} = 2 - \frac{\sqrt{3}(x+y)}{x-y}$$

**45. (b) :** length of tower $AC = \sqrt{h^2+a^2}$

$$= \sqrt{\left(\frac{x-y}{2\sqrt{3}}\right)^2 + \left(\frac{(x-y)(2+\sqrt{3})-2\sqrt{3}x}{2\sqrt{3}}\right)^2}$$

$$= \sqrt{\left(\frac{x-y}{2\sqrt{3}}\right)^2 + \left(\frac{2(x-y)-\sqrt{3}(x+y)}{2\sqrt{3}}\right)^2}$$

$$= \frac{x-y}{2\sqrt{3}}\sqrt{1+\left\{2-\frac{\sqrt{3}(x+y)}{(x-y)}\right]^2}$$

**46. (b) :** We have, $\operatorname{cosec}\left(-\dfrac{73\pi}{3}\right) = -\operatorname{cosec}\left(\dfrac{73\pi}{3}\right)$

$$= -\operatorname{cosec}\left(25\pi - \frac{2\pi}{3}\right) = -\operatorname{cosec}\frac{2\pi}{3} = -\operatorname{cosec}\left(\pi - \frac{\pi}{3}\right)$$

$$= -\operatorname{cosec}\frac{\pi}{3} = -\frac{2}{\sqrt{3}}$$

**47. (a) :** $\cos\left(\dfrac{5\pi}{17}\right) + \cos\left(\dfrac{7\pi}{17}\right) + 2\cos\left(\dfrac{11\pi}{17}\right)\cdot\cos\left(\dfrac{\pi}{17}\right)$

$$= 2\cos\left(\frac{6\pi}{17}\right)\cdot\cos\left(\frac{\pi}{17}\right) + 2\cos\left(\frac{11\pi}{17}\right)\cdot\cos\left(\frac{\pi}{17}\right)$$

$$= 2\cos\left(\frac{6\pi}{17}\right)\cdot\cos\left(\frac{\pi}{17}\right) + 2\cos\left(\pi - \frac{6\pi}{17}\right)\cdot\cos\left(\frac{\pi}{17}\right)$$

$$= 2\cos\left(\frac{6\pi}{17}\right)\cdot\cos\left(\frac{\pi}{17}\right) - 2\cos\left(\frac{6\pi}{17}\right)\cdot\cos\left(\frac{\pi}{17}\right) = 0$$

**48. (b)**

**49. (c) :** $\tan^{-1}\cot(\operatorname{cosec}^{-1} 2) = \tan^{-1}\cot(\cot^{-1} 1)$
$= \tan^{-1}(1) = \pi/4$

**50. (a)** **51. (c)** **52. (b)** **53. (b)**
**54. (a)** **55. (d)** **56. (c)** **57. (b)**
**58. (a)** **59. (c)** **60. (a)** **61. (c)**
**62. (d)** **63. (b)** **64. (c)** **65. (c)**
**66. (d)** **67. (c)** **68. (d)** **69. (a)**
**70. (d)** **71. (a)** **72. (a)** **73. (b)**
**74. (b)** **75. (a)** **76. (c)** **77. (c)**
**78. (b)** **79. (b)** **80. (c)** **81. (c)**
**82. (b)** **83. (a)** **84. (d)** **85. (a)**
**86. (d)** **87. (c)** **88. (c)** **89. (b)**
**90. (d)** **91. (a)** **92. (b)** **93. (b)**
**94. (b)** **95. (d)** **96. (a)** **97. (b)**
**98. (b)** **99. (b)** **100. (c)** **101. (c)**
**102. (b)** **103. (a)** **104. (b)** **105. (c)**
**106. (b)** **107. (b)** **108. (a)** **109. (d)**
**110. (c)** **111. (c)** **112. (d)** **113. (a)**
**114. (c)** **115. (b)** **116. (b)** **117. (a)**
**118. (c)** **119. (d)** **120. (b)**

# Are you ready for Olympiads?

Exam on

**10th Nov, 22nd Nov & 13th Dec 2022**

SOF
SCIENCE OLYMPIAD FOUNDATION

IMO
SOF INTERNATIONAL MATHEMATICS OLYMPIAD

## CLASS XI

### SYLLABUS*

**Section – 1 :** Verbal and Non-Verbal Reasoning.

**Section – 2 :** Sets, Relations and Functions, Principle of Mathematical Induction, Logarithms, Complex Numbers & Quadratic Equations, Linear Inequations, Sequences and Series, Trigonometry, Straight Lines, Conic Sections, Permutations and Combinations, Binomial Theorem, Statistics, Mathematical Reasoning, Limits and Derivatives, Probability, Introduction to 3-D Geometry.

Total Questions : 50

Time : 1 hr.

PATTERN & MARKING SCHEME

| Section | (1) Logical Reasoning | (2) Mathematical Reasoning or Applied Mathematics | (3) Everyday Mathematics | (4) Achievers Section |
|---|---|---|---|---|
| No. of Questions | 15 | 20 | 10 | 5 |
| Marks per Ques. | 1 | 1 | 1 | 3 |

**OR**

**Section – 2 :** Numbers, Quantification, Numerical Applications, Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Mathematical Reasoning, Limits, Continuity and Differentiability, Probability, Descriptive Statistics, Basics of Financial Mathematics, Straight Lines, Circles.

**Section – 3 :** The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

**Section – 4 :** Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Limits and Derivatives, Straight Lines, Circles, Probability.

## Practice Questions

**1.** Find the equations of the line whose sum and product of intercepts on the coordinate axes are –1 and –12 respectively.

A. $4x - 3y - 12 = 0;\ 3x - 4y + 12 = 0$
B. $3x - 4y + 12 = 0;\ x + y - 12 = 0$
C. $4x + 3y + 12 = 0;\ 3x + 4y - 12 = 0$
D. $4x - 3y + 12 = 0;\ 2x + 3y - 12 = 0$

**2.** The two points on the line $x + y = 4$ that lie at a unit distance from the line $4x + 3y = 10$ are

A. $(-3, 1), (7, 11)$ B. $(3, 1), (-7, 11)$
C. $(3, 1), (7, 11)$ D. $(-7, 1), (1, -3)$

**3.** If $z = a + ib$ satisfies $\arg(z - 1) = \arg(z + 3i)$, then $(a - 1) : b =$

A. 2 : 1 B. 1 : 3
C. –1 : 3 D. None of these

**4.** A circle of radius $r$ passes through the origin $O$ and cuts the axes at $A$ and $B$. If the locus of the foot of the perpendicular from $O$ to $AB$ is $(x^2+y^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=\lambda r^2$, then the value of $\lambda$ is

A. 1 B. 2 C. 4 D. 8

**5.** An urn contains 9 balls, 2 of which are white, 3 blue and 4 black. 3 balls are drawn at random from the urn. The chance that 2 balls will be of the same colour and the third of a different colour is

A. $\frac{45}{84}$ B. $\frac{55}{84}$ C. $\frac{35}{84}$ D. $\frac{25}{84}$

**6.** If the function $g(x)$ is defined by

$$g(x)=\frac{x^{200}}{200}+\frac{x^{199}}{199}+\frac{x^{198}}{198}+.....+\frac{x^2}{2}+x+5,$$ then $g'(0) =$

A. 200 B. 5 C. 1 D. 100

**7.** If $S_1$, $S_2$, $S_3$ are the sum of $n$ terms of three A.P.s, the first term of each being unity and the respective common difference being 1, 2, 3, then $S_1 + S_3 =$ ________.

A. $3\,S_2$ B. $4\,S_2$ C. $2\,S_2$ D. $S_2$

**8.** $AB$ is a chord of the parabola $y^2 = 4ax$ with vertex at $A$, $BC$ is drawn perpendicular to $AB$ meeting the axis at $C$. The projection of $BC$ on the $x$-axis is

A. $a$ B. $2a$
C. $8a$ D. None of these

**9.** Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Then the number of ways in which the sitting arrangements can be made is

A. ${}^{11}C_5 \times 9! \times 9!$ B. ${}^{11}P_5 \times 9!$
C. $9! \times 9!$ D. None of these

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**10.** Let $m$ and $n$ be two positive integers greater than 1. If $\lim\limits_{\alpha \to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\frac{e}{2}$, then the value of $\frac{m}{n}$ is

A. 2 B. 4 C. 6 D. 9

**11.** Two finite sets have $m$ and $n$ elements. The total number of subsets of first set is 112 more than the total number of subsets of the second set. Find the value of $m + n$.

A. 11 B. 7

C. 4 D. None of these

**12.** Find the domain of the function,

$f(x) = \sqrt{3-x}+\frac{1}{\sqrt{x^2-1}}$.

A. $x < 3$ B. $x \in (-\infty, -1) \cup (1, 3]$

C. $x \in (-1, 1)$ D. None of these

**13.** Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

A. 10 B. 10.24

C. 11 D. None of these

**14.** A five digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number is divisible by 4.

A. $\frac{2}{5}$ B. $\frac{3}{5}$ C. $\frac{1}{5}$ D. $\frac{1}{2}$

**15.** Find the value of $\left(\sqrt{3}+\sqrt{2}\right)^6+\left(\sqrt{3}-\sqrt{2}\right)^6$.

A. 485 B. 980

C. 1940 D. None of these

## ACHIEVERS SECTION

**16.** Match the linear inequations with their solution set.

| Column-I | | Column-II | |
|---|---|---|---|
| (P) | $3x + 9 \geq -x + 19$ | (1) | $\left(-\frac{17}{25}, -\frac{3}{8}\right)$ |
| (Q) | $\frac{x}{5}<\frac{3x-2}{4}-\frac{5x-3}{5}$ | (2) | $[-7, \infty)$ |
| (R) | $x-2 \leq \frac{5x+8}{3}$ | (3) | $\left[\frac{5}{2}, \infty\right)$ |
| (S) | $\frac{7x-5}{8x+3} > 4$ | (4) | $\left(-\infty, \frac{2}{9}\right)$ |

A. P-1, Q-2, R-3, S-4 B. P-2, Q-1, R-3, S-4

C. P-3, Q-4, R-2, S-1 D. P-2, Q-1, R-4, S-3

**17.** Which of the following statements is incorrect?

A. The domain of the function $f(x)=\sqrt{x-[x]}$ is $R^+$

B. The function $f$ and $g$ are said to be equal, if their domains, ranges are equal and $f(x) = g(x)\ \forall\ x$ in the domain.

C. If $n$ geometric means are inserted between two quantities, then the product of $n$ geometric means is the $n^{\text{th}}$ power of the single geometric mean between the two quantities.

D. The number of permutations of $n$ different objects taken $r$ at a time, where repetition is allowed is $n^r$.

**18.** **Statement-I :** The points $A(3, -1, 2)$, $B(1, 2, -4)$, $C(-1, 1, 2)$ and $D(1, -2, 8)$ are the vertices of a parallelogram.

**Statement-II :** Coordinates of mid-point of a line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

A. Both Statement-I and Statement-II are true.

B. Statement-I is true but Statement-II is false.

C. Statement-I is false but Statement-II is true.

D. Both Statement-I and Statement-II are false.

**19.** Two dice are thrown. The events $P$, $Q$ and $R$ are as follows :

$P$ : getting an even number on the first die.

$Q$ : getting an odd number on the first die.

$R$ : getting the sum of the numbers on the dice $\leq 5$.

Then, which of the following is true?

(i) $P$ and $Q$ are mutually exclusive.

(ii) $P$ and $Q$ are mutually exclusive and exhaustive.

(iii) $P = Q'$

(iv) $P$ and $R$ are mutually exclusive.

(v) $P'$, $Q'$ and $R$ are mutually exclusive and exhaustive.

A. (i), (iv), (v) B. (ii), (iv), (v)

C. (iii), (v) D. (i), (ii) and (iii)

**20.** Evaluate : $\lim\limits_{x \to 0}\frac{\tan x+4\tan 2x-3\tan 3x}{x^2\tan x}$

A. $-16$ B. 16 C. 8 D. $-8$

Darken your choice with HB Pencil

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | Ⓐ Ⓑ Ⓒ Ⓓ | **5.** | Ⓐ Ⓑ Ⓒ Ⓓ | **9.** | Ⓐ Ⓑ Ⓒ Ⓓ | **13.** | Ⓐ Ⓑ Ⓒ Ⓓ | **17.** | Ⓐ Ⓑ Ⓒ Ⓓ |
| **2.** | Ⓐ Ⓑ Ⓒ Ⓓ | **6.** | Ⓐ Ⓑ Ⓒ Ⓓ | **10.** | Ⓐ Ⓑ Ⓒ Ⓓ | **14.** | Ⓐ Ⓑ Ⓒ Ⓓ | **18.** | Ⓐ Ⓑ Ⓒ Ⓓ |
| **3.** | Ⓐ Ⓑ Ⓒ Ⓓ | **7.** | Ⓐ Ⓑ Ⓒ Ⓓ | **11.** | Ⓐ Ⓑ Ⓒ Ⓓ | **15.** | Ⓐ Ⓑ Ⓒ Ⓓ | **19.** | Ⓐ Ⓑ Ⓒ Ⓓ |
| **4.** | Ⓐ Ⓑ Ⓒ Ⓓ | **8.** | Ⓐ Ⓑ Ⓒ Ⓓ | **12.** | Ⓐ Ⓑ Ⓒ Ⓓ | **16.** | Ⓐ Ⓑ Ⓒ Ⓓ | **20.** | Ⓐ Ⓑ Ⓒ Ⓓ |

## SOLUTIONS

**1. (A) :** Let the equation in intercept form be $\frac{x}{a}+\frac{y}{b}=1,$ where, $x$-intercept = $a$ and $y$-intercept = $b$

According to question, $a + b = -1$ ...(i)

and $ab = -12$ ...(ii)

∴ $(a - b)^2 = (a + b)^2 - 4ab = (-1)^2 - 4(-12) = 49 = 7^2$

$\Rightarrow a - b = 7$ ...(iii)

and $a - b = -7$ ...(iv)

Adding (i) and (iii), we get $2a = 6 \Rightarrow a = 3$

Putting $a = 3$ in (i), we get $3 + b = -1 \Rightarrow b = -4$

∴ The required equation is $\frac{x}{3}+\frac{y}{-4}=1 \Rightarrow \frac{x}{3}-\frac{y}{4}=1$

$\Rightarrow 4x - 3y = 12$ or $4x - 3y - 12 = 0$

Adding (i) and (iv), we get $2a = -8 \Rightarrow a = -4$

Putting $a = -4$ in (i), we get $b = 3$

∴ The required equation is $-\frac{x}{4}+\frac{y}{3}=1$

$\Rightarrow 3x - 4y + 12 = 0$

**2. (B) :** We have, $x + y = 4$ ....(i)

$4x + 3y = 10$ ....(ii)

Let any point on the line (i) be $(t, 4 - t)$

Now, $\frac{|4t+3(4-t)-10|}{\sqrt{(4)^2+(3)^2}}=1 \Rightarrow |t+2|=5$

$\Rightarrow t + 2 = \pm 5 \Rightarrow t = -7, 3$

∴ Required points are $(-7, 11)$ and $(3, 1)$.

**3. (B) :** We have, $\arg(z - 1) = \arg(z + 3i)$

Put $z = a + ib$

∴ $\arg(a + ib - 1) = \arg(a + ib + 3i)$

$\Rightarrow \arg[(a - 1) + ib] = \arg[a + i(3 + b)]$

$\Rightarrow \tan^{-1}\left(\frac{b}{a-1}\right)=\tan^{-1}\left(\frac{3+b}{a}\right)$

Now, $\frac{b}{a-1}=\frac{3+b}{a} \Rightarrow ab=3a+ab-3-b$

$\Rightarrow 3a - 3 = b \Rightarrow 3(a - 1) = b \Rightarrow (a - 1) : b = 1 : 3$

**4. (C) :** The equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$, where,

Centre $\equiv (-g, -f)$, and radius, $r=\sqrt{g^2+f^2-c}$

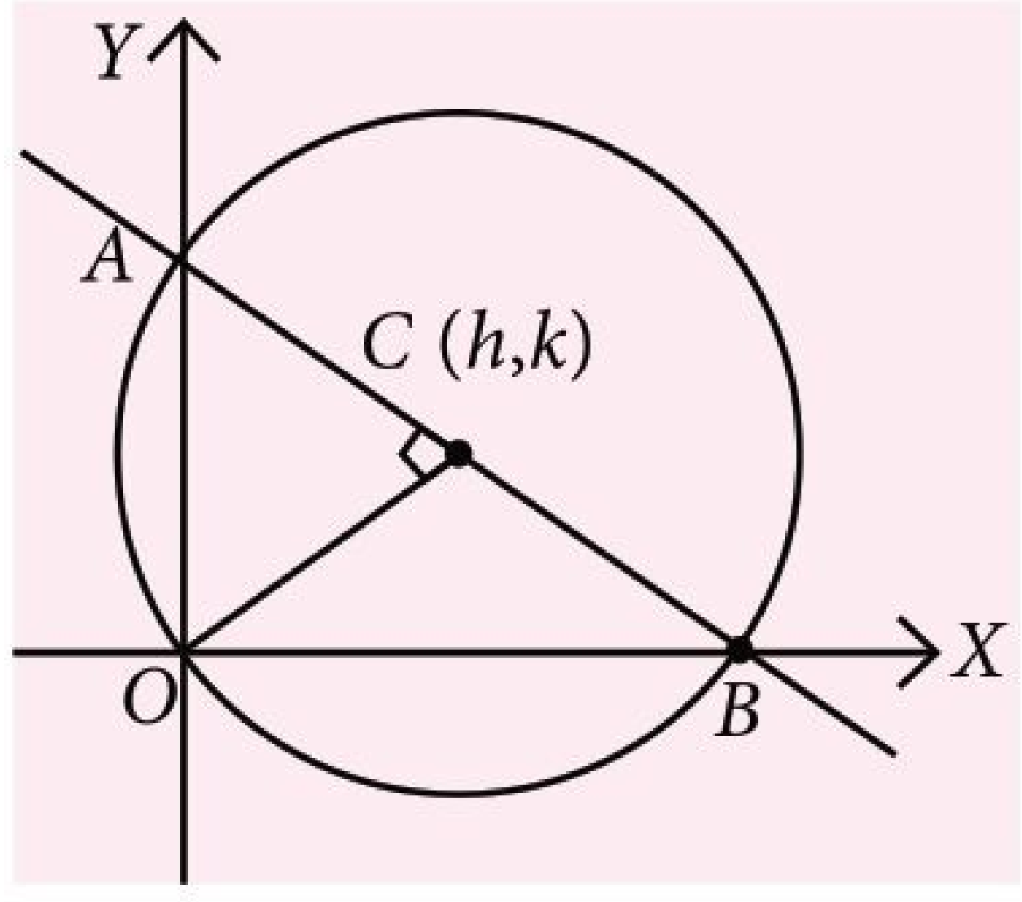
Let $C \equiv (h,k)$

Equation of the line $AB$ is

$\frac{y-k}{x-h}=-\frac{h}{k} \Rightarrow yk-k^2=h^2-hx$

$\Rightarrow yk + hx = h^2 + k^2$

∴ $A\left(0, \frac{h^2+k^2}{k}\right)$ and $B\left(\frac{h^2+k^2}{h}, 0\right)$



Now, $AB = 2r$

$\left(\frac{h^2+k^2}{h}\right)^2+\left(\frac{h^2+k^2}{k}\right)^2=4r^2$

$(h^2+k^2)^2\left(\frac{1}{h^2}+\frac{1}{k^2}\right)=4r^2$

∴ The locus is $(x^2+y^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=4r^2$

Hence, $\lambda = 4$.

**5. (B) :** We have,

2 white, 3 blue, 4 black balls.

Required probability

$=\frac{{}^2C_2\times{}^7C_1}{{}^9C_3}+\frac{{}^3C_2\times{}^6C_1}{{}^9C_3}+\frac{{}^4C_2\times{}^5C_1}{{}^9C_3}=\frac{55}{84}$

**6. (C) :** We have

$g(x)=\frac{x^{200}}{200}+\frac{x^{199}}{199}+\frac{x^{198}}{198}+\ldots+\frac{x^2}{2}+x+5$

$g'(x)=\frac{200x^{199}}{200}+\frac{199x^{198}}{199}+\frac{198x^{197}}{198}+\ldots+\frac{2x}{2}+1$

Put $x = 0$

$g'(0) = 1$.

**7. (C) :** For first A.P.

First term = 1, common difference = 1

∴ $S_1 = 1 + 2 + 3 + \ldots\ldots\ n$ terms.

$=\frac{n(n+1)}{2}$

For second A.P.

First term = 1, common difference = 2

$S_2 = 1 + 3 + 5 + 7 + \ldots\ldots\ n$ terms.

$$S_2=\frac{n}{2}[2+(n-1)2]=n^2$$

For third A.P.

First term = 1, common difference = 3

$S_3$ = 1 + 4 + 7 + 10 +...... $n$ terms.

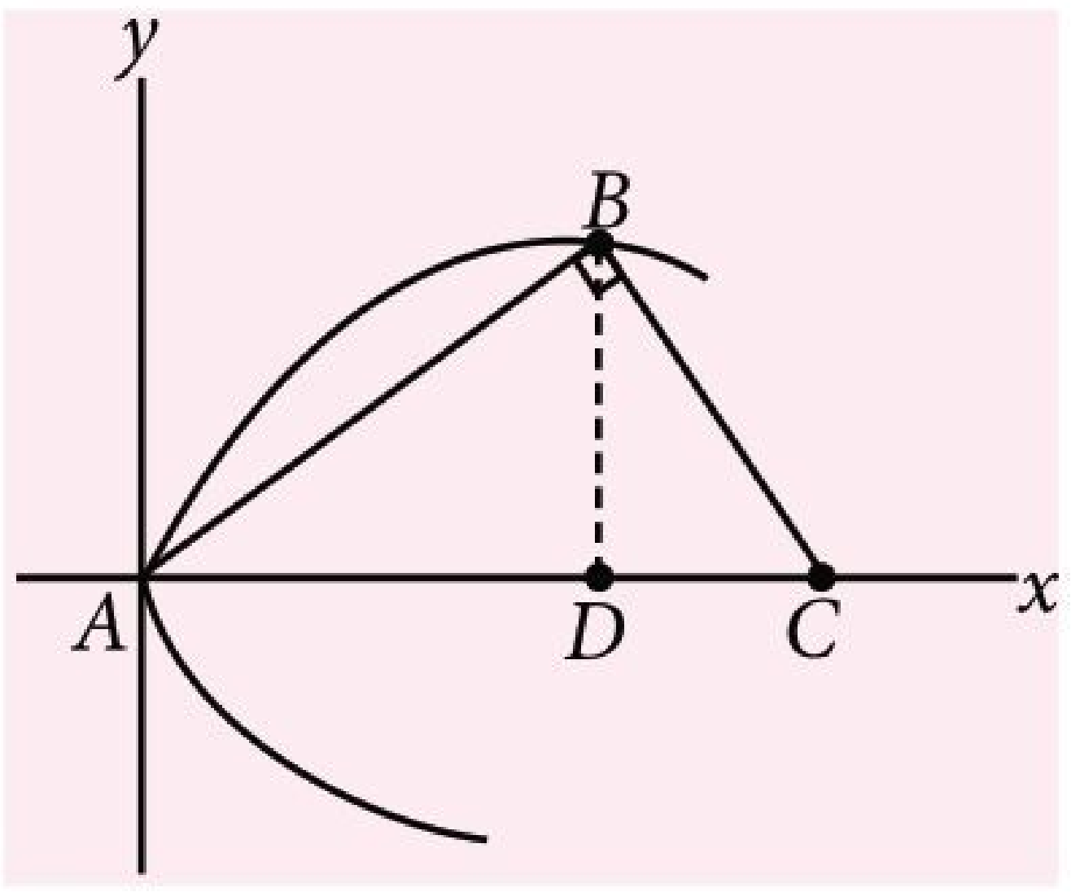
$$S_3=\frac{n}{2}[2+(n-1)3]$$

$$=\frac{n}{2}(3n-1)$$

$$\text{Now, } S_1+S_3=\frac{n(n+1)}{2}+\frac{n}{2}(3n-1)=2n^2$$

$= 2S_2$

**8. (D) :** Let $B \equiv (at^2, 2at)$



Slope of $AB=\frac{2}{t}$

$\therefore$ Slope of $BC=-\frac{t}{2}$

Equation of $BC:\frac{y-2at}{x-at^2}=-\frac{t}{2}$

$\Rightarrow$ $2y - 4at = at^3 - xt$

$\because$ $BC$ intersect at point $C$ on $x$–axis. So, putting $y = 0$ in above equation, we get

$x = 4a + at^2$

$\therefore$ $C \equiv (4a + at^2, 0)$

Now $CD = 4a + at^2 - at^2 = 4a$.

**9. (A) :** Out of 18 guests, 4 particular guests desire to sit on one particular side $A$ and other 3 on other side let say $B$.

Out of 18 – 4 – 3 = 11 guests, we have to select 5 more guests for side A and 6 more guests for side $B$.

Out of 11 guests, 5 guests can be selected in ${}^{11}C_5$ ways.

We have permute 9 men among themselves in 9! ways in each side.

$\therefore$ The required ways = ${}^{11}C_5\times(9!)^2$

**10. (A) :** We have, $\lim\limits_{\alpha\to0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\frac{e}{2}$

$$\Rightarrow \lim_{\alpha\to0}\left(\frac{e^{\cos(\alpha^n)-1}-1}{\cos(\alpha^n)-1}\cdot\frac{\cos(\alpha^n)-1}{\alpha^m}\cdot e\right)=-\frac{e}{2}$$

$$\Rightarrow \lim_{\alpha\to0}\left(\frac{1-\cos\alpha^n}{\alpha^m}\right)=\frac{1}{2}$$

$$\Rightarrow \lim_{\alpha\to0}\left(\frac{1-\cos\alpha^n}{\left(\alpha^n\right)^2}\cdot\alpha^{2n-m}\right)=\frac{1}{2}\Rightarrow 2n-m=0\Rightarrow\frac{m}{n}=\frac{2}{1}$$

**11. (A) :** Let first set have $m$ elements. Then, according to given condition, we have

$2^m = 112 + 2^n$

[$\because$ Total number of subsets of a finite set containing $k$ elements = $2^k$]

$\Rightarrow$ $2^m - 2^n = 112$

$\Rightarrow$ $2^n(2^{m-n} - 1) = 112 = 16 \times 7 = 2^4(2^3 - 1)$

$\Rightarrow$ $n = 4$ and $m - n = 3$

$\Rightarrow$ $m = 7$ and $n = 4$

**12. (B) :** We have, $f(x)=\sqrt{3-x}+\frac{1}{\sqrt{x^2-1}}$

Clearly, $f(x)$ is defined for all real values of $x$ for which

$3 - x \ge 0$ and $x^2 - 1 > 0$

$\Rightarrow$ $x - 3 \le 0$ and $(x - 1)(x + 1) > 0$

$\Rightarrow$ $x \le 3$ and ($x < -1$ or $x > 1$)

$\Rightarrow$ ($x \le 3$ and $x < -1$) or ($x \le 3$ and $x > 1$)

$\Rightarrow$ $x < -1$ or $1 < x \le 3$ $\Rightarrow$ $x \in (-\infty, -1) \cup (1, 3]$

**13. (B) :** We have given, $n$ = 100,

Incorrect mean $=\bar{x}=40$, Incorrect S.D. $=\sigma=10$

$$\therefore \frac{\Sigma x_i}{n}=40 \Rightarrow \frac{\Sigma x_i}{100}=40$$

$\Rightarrow$ Incorrect $\Sigma x_i = 4000$

Hence, correct $\Sigma x_i$ = 4000 – 30 – 70 + 3 + 27 = 3930

$$\therefore \text{Correct mean } =\frac{3930}{100}=39.3$$

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Also, $\sigma^2 = \frac{\Sigma x_i^2}{n} - (40)^2 \Rightarrow \quad 100 = \frac{\Sigma x_i^2}{100} - 1600$

$\Rightarrow$ Incorrect $\Sigma x_i^2 = 170000$

Now, correct $\Sigma x_i^2$

$= 170000 - (30)^2 - (70)^2 + 3^2 + (27)^2 = 164938$

$\therefore$ Correct S.D. $= \sqrt{\frac{164938}{100} - (39.3)^2}$

$= \sqrt{1649.38 - 1544.49} = \sqrt{104.89} = 10.24$

**14. (C) :** Total number of five digit numbers formed by the digits 1, 2, 3, 4, 5 without repetition is 5! = 120

We know that a number is divisible by 4, if the number formed by last two digits is divisible by 4. Here, last two digits can be 12, 24, 32, 52, *i.e.*, last two digits can be filled in 4 ways. But corresponding to each of these ways there are 3! = 6 ways of filling the remaining three places.

Therefore, the total number of five digit numbers formed by the digits 1, 2, 3, 4, 5 and divisible by 4 = 4 × 6 = 24

So, required probability $= \frac{24}{120} = \frac{1}{5}$

**15. (D) :** Clearly

$$\left(\sqrt{3}+\sqrt{2}\right)^6 = {}^6C_0\left(\sqrt{3}\right)^6 + {}^6C_1\left(\sqrt{3}\right)^5\left(\sqrt{2}\right)^1 + {}^6C_2\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^2 + {}^6C_3\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3 + {}^6C_4\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^4 + {}^6C_5\left(\sqrt{3}\right)^1\left(\sqrt{2}\right)^5 + {}^6C_6\left(\sqrt{2}\right)^6 \quad \ldots\text{(i)}$$

and $$\left(\sqrt{3}-\sqrt{2}\right)^6 = {}^6C_0\left(\sqrt{3}\right)^6 - {}^6C_1\left(\sqrt{3}\right)^5\left(\sqrt{2}\right)^1 + {}^6C_2\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^2 - {}^6C_3\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3 + {}^6C_4\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^4 - {}^6C_5\left(\sqrt{3}\right)\left(\sqrt{2}\right)^5 + {}^6C_6\left(\sqrt{2}\right)^6 \quad \ldots\text{(ii)}$$

On adding (i) and (ii), we get

$$\left(\sqrt{3}+\sqrt{2}\right)^6 + \left(\sqrt{3}-\sqrt{2}\right)^6 = 2[{}^6C_0\left(\sqrt{3}\right)^6 + {}^6C_2\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^2 + {}^6C_4\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^4 + {}^6C_6\left(\sqrt{2}\right)^6]$$

$$= 2\left[1\cdot 3^3 + \frac{6\times 5}{2\times 1}\cdot 3^2\cdot 2 + \frac{6\times 5}{2\times 1}\cdot 3\cdot 2^2 + 1\cdot 2^3\right]$$

$= 2[27 + 270 + 180 + 8]$

$= 2(485) = 970$

**16. (C) : (P)** $3x + 9 \geq -x + 19 \Rightarrow 4x \geq 10$

$\Rightarrow x \geq \frac{5}{2} \Rightarrow x \in \left[\frac{5}{2}, \infty\right).$

**(Q)** $\frac{x}{5} < \frac{3x-2}{4} - \frac{5x-3}{5}$

$\Rightarrow \frac{x}{5} < \frac{5(3x-2) - 4(5x-3)}{20} \Rightarrow 4x < -5x + 2$

$\Rightarrow 9x < 2 \Rightarrow x < \frac{2}{9} \Rightarrow x \in \left(-\infty, \frac{2}{9}\right).$

**(R)** $x - 2 \leq \frac{5x+8}{3} \Rightarrow 3x - 6 \leq 5x + 8$

$\Rightarrow 2x \geq -14 \Rightarrow x \geq -7 \quad \Rightarrow \quad x \in [-7, \infty).$

**(S)** $\frac{7x-5}{8x+3} > 4 \Rightarrow \frac{7x-5}{8x+3} - 4 > 0$

$\Rightarrow \frac{-25x-17}{8x+3} > 0 \quad \Rightarrow \frac{25x+17}{8x+3} < 0$

| | |
|---|---|
| $\Rightarrow$ Either $25x + 17 > 0$ and $8x + 3 < 0$ | or $25x + 17 < 0$ and $8x + 3 > 0$ |
| $\Rightarrow x > \frac{-17}{25}$ and $x < \frac{-3}{8}$ | $\Rightarrow x < \frac{-17}{25}$ and $x > \frac{-3}{8}$ |
| $\Rightarrow \frac{-17}{25} < x < \frac{-3}{8}$ | which is not possible. |

$\therefore \quad x \in \left(\frac{-17}{25}, \frac{-3}{8}\right).$

Thus, (P) $\rightarrow$ 3; (Q) $\rightarrow$ 4; (R) $\rightarrow$ 2 ; (S) $\rightarrow$ 1

**17. (A) :** A. $f(x) = \sqrt{x - [x]}$

$f(x)$ will be defined for $x - [x] \geq 0 \Rightarrow x \geq [x]$.

which is true for all $x \in R$.

$\therefore$ Domain of $f(x)$ is $R$.

Clearly, *B*, *C* and *D* are true.

**18. (A) :** Mid-point of *AC* $= \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right)$

$= (1, 0, 2)$

Mid-point of *BD* $= \left(\frac{1+1}{2}, \frac{2-2}{2}, \frac{-4+8}{2}\right) = (1, 0, 2)$

$\because$ Mid-points of *AC* and *BD* coincides.

$\therefore$ *ABCD* is a parallelogram.

**19. (D) :** *P* = Getting an even number on the first die

= {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

*Q* = Getting an odd number on the first die

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

$R$ = Getting the sum of numbers $\leq 5$
$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$

Now, $P \cap Q = \phi$

$\therefore$ $P$ and $Q$ are mutually exclusive events.

So, (i) is true.

Also, $P \cup Q = S \Rightarrow P$ and $Q$ are mutually exhaustive.

So, (ii) is true.

Now, $Q$ = Getting an odd number on the first die

$\Rightarrow$ $Q'$ = Getting an even number on the first die = $P$

$\therefore$ $P = Q'$.

So, (iii) is true.

Now, $P \cap R = \{(2, 1), (2, 2), (2, 3), (4, 1)\} \neq \phi$

$\Rightarrow$ $P$ and $R$ are not mutually exclusive.

So, (iv) is false.

Now, $P' \cap Q' = \phi$ $\quad (\because P = Q')$

$\therefore$ $P' \cap Q' \cap R = \phi$ and $P' \cup Q' \cup R = S$

But $P' \cap R = Q \cap R$ $\quad (\because P' = Q)$

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)\} \neq \phi$

$\therefore$ $P'$, $Q'$ and $R$ are not mutually exclusive.

So, (v) is false.

**20. (A) :** We have, $\lim\limits_{x\to 0} \dfrac{\tan x + 4\tan 2x - 3\tan 3x}{x^2 \tan x}$

$$= \lim_{x\to 0} \frac{\tan x + 4\left(\dfrac{2\tan x}{1-\tan^2 x}\right) - 3\left(\dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}\right)}{x^2 \tan x}$$

$$= \lim_{x\to 0} \frac{1 + \dfrac{8}{1-\tan^2 x} - 3\left(\dfrac{3-\tan^2 x}{1-3\tan^2 x}\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{(1-\tan^2 x)(1-3\tan^2 x) + 8(1-3\tan^2 x) - 3(3-\tan^2 x)(1-\tan^2 x)}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$= \lim_{x\to 0} \frac{1 - 4\tan^2 x + 3\tan^4 x + 8 - 24\tan^2 x - 9 + 12\tan^2 x - 3\tan^4 x}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$= \lim_{x\to 0} \frac{-16\tan^2 x}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$= -16 \lim_{x\to 0} \left(\frac{\tan x}{x}\right)^2 \times \frac{1}{1-\tan^2 x} \times \frac{1}{1-3\tan^2 x} = -16$$

*For other sections/subjects please refer to Physics For You, Chemistry Today and Biology Today*

◆◆

## UNSCRAMBLE ME

**Unscramble the words given in column I and match them with their explanations in column II.**

| Column I | Column II |
|---|---|
| 1. GRONITEATIN | (a) A function, in which every element in the co-domain has at least one corresponding element in the domain. |
| 2. SPEILEL | (b) Transpose of a co-factor matrix. |
| 3. NEATIOQU | (c) A process of addition of infinitesimal small values on a large scale, when addition can not be performed. |
| 4. STECRIJUEV | (d) The locus of all the points in a plane such that the sum of their distances from two fixed points is constant. |
| 5. JATIONSD | (e) A line that touches the curve or a circle at a point. |
| 6. ANENTGT | (f) A mathematical statement made up of two expressions joined by an equal sign. |
| 7. CINRELIC | (g) The point of intersection of external angle bisectors opposite to a vertex. |
| 8. ENTRXEEC | (h) The locus of the middle points of a system of parallel chords of a conic section. |
| 9. MOETTPSYA | (i) A straight line that constantly approaches a certain curve but does not meet at any infinite distance. |
| 10. ATEMERID | (j) The circle touching the three sides of a triangle internally. |

*Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names will be published in next issue.*

# Are you ready for Olympiads?

**Exam on**

**10th Nov, 22nd Nov & 13th Dec 2022**

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## CLASS XII

### SYLLABUS*

**Section – 1 :** Verbal and Non-Verbal Reasoning.

**Section – 2 :** Relations and Functions, Inverse Trigonometric Functions, Matrices and Determinants, Continuity and Differentiability, Application of Derivatives, Integrals, Application of Integrals, Differential Equations, Vector Algebra, Three Dimensional Geometry, Probability, Linear Programming.

**OR**

**Section – 2 :** Numbers, Quantification, Numerical Applications, Solutions of Simultaneous Linear Equations, Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Probability, Inferential Statistics, Index numbers, Time-based data, Financial Mathematics, Linear Programming.

**Section – 3 :** The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

**Section – 4 :** Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Linear Programming, Probability.

Total Questions : 50

Time : 1 hr.

**PATTERN & MARKING SCHEME**

| Section | (1) Logical Reasoning | (2) Mathematical Reasoning or Applied Mathematics | (3) Everyday Mathematics | (4) Achievers Section |
|---|---|---|---|---|
| No. of Questions | 15 | 20 | 10 | 5 |
| Marks per Ques. | 1 | 1 | 1 | 3 |

## Practice Questions

1. The value of $\int e^x\left[\frac{x^3+x+1}{(1+x^2)^{3/2}}\right]dx$ equals

   A. $e^x\frac{x}{\sqrt{1+x^2}}+C$   B. $e^x\cdot\frac{e^x}{\sqrt{1+x^2}}+C$

   C. $e^x+\frac{x}{\sqrt{1+x^2}}+C$   D. $e^x\sqrt{1+x^2}+C$

2. The system of linear equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$ has no solution when

   A. $a = 2, b \neq 3$   B. $b = 2, a = 3$

   C. $a = 3, b \neq 10$   D. $b = 3, a \neq 10$

3. The solution of differential equation

   $x^2y-x^3\frac{dy}{dx}=y^4\cos x$ is

   A. $x^2y^{-3} = 2\sin x + c$   B $x^2y^{-3} = 3\cos x + c$

   C. $x^3y^{-3} = 3\sin x + c$   D. $x^2y^3 = 3\sin x + c\,x^2y$

4. A box contains $N$ coins, $m$ of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. First time it shows head and second time it shows tail. The probability that the drawn coin is fair is

   A. $\frac{8m}{m+9N}$   B. $\frac{9m}{8m+N}$

   C. $\frac{9m}{m+8N}$   D. None of these

5. The area (in square units) bounded by the curves $y=\sqrt{x}, 2y-x+3=0$, $x$-axis and lying in the first quadrant, is

   A. 9   B. 36   C. 18   D. 27/4

6. Find the shortest distance between the line $y - x = 1$ and the curve $x = y^2$.

   A. $\frac{3}{8}$   B. $\frac{3\sqrt{2}}{8}$

   C. $\frac{3\sqrt{2}}{4}$   D. None of these

7. The maximum value of $z = x + y$ subject to $x + y \leq 3$, $-2x + y \leq 1$, $x \leq 2$, $x, y \geq 0$ is

   A. 2   B. 3   C. 4   D. 5

8. If $g(x) = x^2 - 4x - 5$, then

   A. $g$ is one-one on $R$

   B. $g$ is one-one on $(-\infty, 2]$

   C. $g$ is one-one on $(-\infty, 4]$

   D. None of these

---

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**9.** The set of all points where the function $f(x) = \sqrt[3]{x^2|x|}$ is differentiable is

A. $[0, \infty)$ B. only $(0, \infty)$
C. $(-\infty, \infty)$ D. $(-\infty, 0) \cup (0, \infty)$

**10.** If $f(x) = \begin{cases} 3x^2+12x-1, & -1 \le x \le 2, \\ 37-x & , 2 < x \le 3 \end{cases}$, then

A. $f(x)$ is increasing in $[-1, 2]$
B. $f(x)$ is continuous in $[-1, 3]$
C. $f'(2)$ does not exist
D. All of these

**11.** Given a function defined by $f(x)=\sqrt{4-x^2}$, $0 \le x \le 2$, $0 \le f(x) \le 2$. Then $f$ is

A. Only one one B. Only onto
C. Bijective function D. None of these

**12.** Evaluate : $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$.

A. $\frac{3+\sqrt{5}}{2}$ B. $\frac{3-\sqrt{5}}{2}$
C. $3+\sqrt{5}$ D. $3-\sqrt{5}$

**13.** If $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then find $\alpha$ satisfying $0<\alpha<\frac{\pi}{2}$, when $A + A' = \sqrt{2}\,I_2$, where $A'$ is transpose of $A$.

A. 0 B. $\frac{3\pi}{4}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$

**14.** Find $p$, if the points $(p + 1, 1)$, $(2p + 1, 3)$ and $(2p + 2, 2p)$ are collinear.

A. 2 B. $\frac{1}{2}$
C. $-2$ D. None of these

**15.** Find the intervals in which $f(x)=\frac{x}{\log x}$ is increasing.

A. $(0, 1)$ B. $(1, e)$
C. $(e, \infty)$ D. None of these

## ACHIEVERS SECTION

**16.** Read the given statements carefully and state 'T' for true and 'F' for false.

(i) The area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ is $\frac{3}{2}(\pi-2)$ sq. units

(ii) The area of the function $y = \sin^2 x$ from 0 to $\pi$ will be more than that of curve $y = \sin x$ from 0 to $\pi$.

(iii) $\int e^x(\tan x+\sec^2 x)\,dx=e^x\cdot\tan x+c$

| | **(i)** | **(ii)** | **(iii)** |
|---|---|---|---|
| A. | T | F | T |
| B. | F | T | F |
| C. | T | T | T |
| D. | F | T | F |

**17.** Fill in the blanks and select the correct option.

(i) The degree of the differential equation $2\frac{d^2y}{dx^2}+3\sqrt{1-\left(\frac{dy}{dx}\right)^2}-y=0$ is **P**.

(ii) The general solution of differential equation $\frac{dy}{dx}=(4x+y+1)^2$ is **Q**.

(iii) The solution of differential equation $\frac{dy}{dx}=e^{x-y}(e^x-e^y)$ is **R**.

| | **(P)** | **(Q)** | **(R)** |
|---|---|---|---|
| A. | 3 | $\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + c$ | $e^x = e^y - 1 + ce^{-e^x}$ |
| B. | 2 | $\tan^{-1}\left(\frac{4x+y+1}{2}\right) = y + c$ | $e^y = e^x - ce^{-e^x}$ |
| C. | 2 | $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + c$ | $e^y = e^x - 1 + ce^{-e^x}$ |
| D. | Not defined | $\frac{1}{2}\tan^{-1}\left(\frac{4x-y-1}{2}\right) = x + c$ | $e^y = e^x - 1$ |

**18.** **Statement-I :** For two events $A$ & $B$ such that $P(A)=P(A|B)=\frac{1}{4}$ & $P(B|A)=\frac{1}{2}$,then $P(\bar{A}|B)=\frac{3}{4}$ & $P(\bar{B}|A)=\frac{1}{2}$.

**Statement-II :** Two events $A$ & $B$ are independent, then $P(B|A) = P(B)$.

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A. Both Statement-I and Statement-II are true.
B. Both Statement-I and Statement-II are false.
C. Statement-I is true but Statement-II is false.
D. Statement-I is false but Statement-II is true.

**19.** Match Column I with Column II and select the correct option.

| | Column I | | Column II |
|---|---|---|---|
| (P) | If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}+2\vec{b}\rvert$, then angle between $\vec{a}$ and $\vec{b}$ is | (1) | 90° |
| (Q) | If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}-2\vec{b}\rvert$, then angle between $\vec{a}$ and $\vec{b}$ is | (2) | obtuse |
| (R) | If $\lvert\vec{a}+\vec{b}\rvert=\lvert\vec{a}-\vec{b}\rvert$, then angle between $\vec{a}$ and $\vec{b}$ is | (3) | 0° |
| (S) | Angle between $\vec{a}\times\vec{b}$ and a vector perpendicular to the vector $\vec{c}\times(\vec{a}\times\vec{b})$ is | (4) | acute |

A. P-1; Q-2; R-3; S-4 B. P-2; Q-4; R-1; S-3
C. P-3; Q-4; R-2; S-1 D. P-2; Q-3; R-1; S-4

**20.** If $P$, $Q$, $R$, $S$ are (3, 6, 4), (2, 5, 2), (6, 4, 4), (0, 2, 1) respectively, find the projection of $PQ$ on $RS$.
A. 1 B. 7
C. 2 D. None of these

**Darken your choice with HB Pencil**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | Ⓐ Ⓑ Ⓒ Ⓓ | 5. | Ⓐ Ⓑ Ⓒ Ⓓ | 9. | Ⓐ Ⓑ Ⓒ Ⓓ | 13. | Ⓐ Ⓑ Ⓒ Ⓓ | 17. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 2. | Ⓐ Ⓑ Ⓒ Ⓓ | 6. | Ⓐ Ⓑ Ⓒ Ⓓ | 10. | Ⓐ Ⓑ Ⓒ Ⓓ | 14. | Ⓐ Ⓑ Ⓒ Ⓓ | 18. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 3. | Ⓐ Ⓑ Ⓒ Ⓓ | 7. | Ⓐ Ⓑ Ⓒ Ⓓ | 11. | Ⓐ Ⓑ Ⓒ Ⓓ | 15. | Ⓐ Ⓑ Ⓒ Ⓓ | 19. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 4. | Ⓐ Ⓑ Ⓒ Ⓓ | 8. | Ⓐ Ⓑ Ⓒ Ⓓ | 12. | Ⓐ Ⓑ Ⓒ Ⓓ | 16. | Ⓐ Ⓑ Ⓒ Ⓓ | 20. | Ⓐ Ⓑ Ⓒ Ⓓ |

## SOLUTIONS

**1. (A) :** We have, $I=\int e^x\left[\frac{x^3+x+1}{(1+x^2)^{3/2}}\right]dx$

$$=\int e^x\left[\frac{x(x^2+1)+1}{(1+x^2)^{3/2}}\right]dx \;=\int e^x\left[\frac{x}{\sqrt{1+x^2}}+\frac{1}{(1+x^2)^{3/2}}\right]dx$$

$$=e^x\cdot\frac{x}{\sqrt{1+x^2}}+C \quad \left(\because\ \int e^x(f(x)+f'(x))dx=e^x f(x)+C\right)$$

**2. (C) :** We have given system of linear equation as:

$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + az = b$

Now, $AX = B$, where

$$A=\begin{bmatrix}1&1&1\\1&2&3\\1&2&a\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}6\\10\\b\end{bmatrix}$$

For no solution, $|A| = 0$ and adj $(A)\cdot B \neq 0$

$$|A|=\begin{vmatrix}1&1&1\\1&2&3\\1&2&a\end{vmatrix}=1(2a-6)-1(a-3)+1(2-2)$$

$= 2a - 6 - a + 3 = 0 \Rightarrow a - 3 = 0 \Rightarrow a = 3$

$$\text{Now, adj}(A)=\begin{bmatrix}0&-1&1\\0&2&-2\\0&-1&1\end{bmatrix}$$

$$\text{adj }(A)\cdot B=\begin{bmatrix}0&-1&1\\0&2&-2\\0&-1&1\end{bmatrix}\begin{bmatrix}6\\10\\b\end{bmatrix}\neq 0$$

$$\Rightarrow \begin{bmatrix}0-10+b\\0+20-2b\\0-10+b\end{bmatrix}\neq 0 \;\Rightarrow\; b\neq 10$$

**3. (C) :** We have, $x^2y-x^3\frac{dy}{dx}=y^4\cos x$

$$\Rightarrow -\frac{1}{y^4}\frac{dy}{dx}+\frac{1}{xy^3}=\frac{\cos x}{x^3} \qquad \text{...(i)}$$

Put $v=\frac{1}{y^3} \Rightarrow \frac{dv}{dx}=\frac{-3}{y^4}\frac{dy}{dx}$ in (i), we get

$$\frac{1}{3}\frac{dv}{dx}+\frac{v}{x}=\frac{\cos x}{x^3} \Rightarrow \frac{dv}{dx}+\frac{3v}{x}=\frac{3\cos x}{x^3}$$

This is a linear D.E. where $Q=\frac{3}{x}, R=\frac{3\cos x}{x^3}$

$\text{I.F.} = e^{\int \frac{3}{x}dx} = e^{\log x^3} = x^3$

∴ Solution is given by

$$v\cdot(x^3) = \int \frac{3\cos x}{x^3}\cdot x^3 dx + c$$

⇒ $v(x^3) = 3\sin x + c$ ⇒ $x^3y^{-3} = 3\sin x + c$

**4. (C) :** Let $E_1$ : Selected coin is fair.

$E_2$ : Selected coin is biased.

$A$ : The first toss results in a head and the second toss result in a tail.

∴ $P(E_1) = \frac{m}{N}, P(E_2) = \frac{N-m}{N}$

For fair coin : $P(\text{head}) = P(\text{Tail}) = \frac{1}{2}$

For biased coin : $P(\text{head}) = \frac{2}{3}$, $P(\text{Tail}) = \frac{1}{3}$

Now, $P(A|E_1) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}, P(A|E_2) = \frac{2}{3}\times\frac{1}{3} = \frac{2}{9}$

∴ Required probability

$$= P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{m}{N}\times\frac{1}{4}}{\frac{m}{N}\times\frac{1}{4} + \frac{N-m}{N}\times\frac{2}{9}} = \frac{m}{4N}\times\frac{36N}{m+8N} = \frac{9m}{m+8N}$$

**5. (A) :** Given curves are

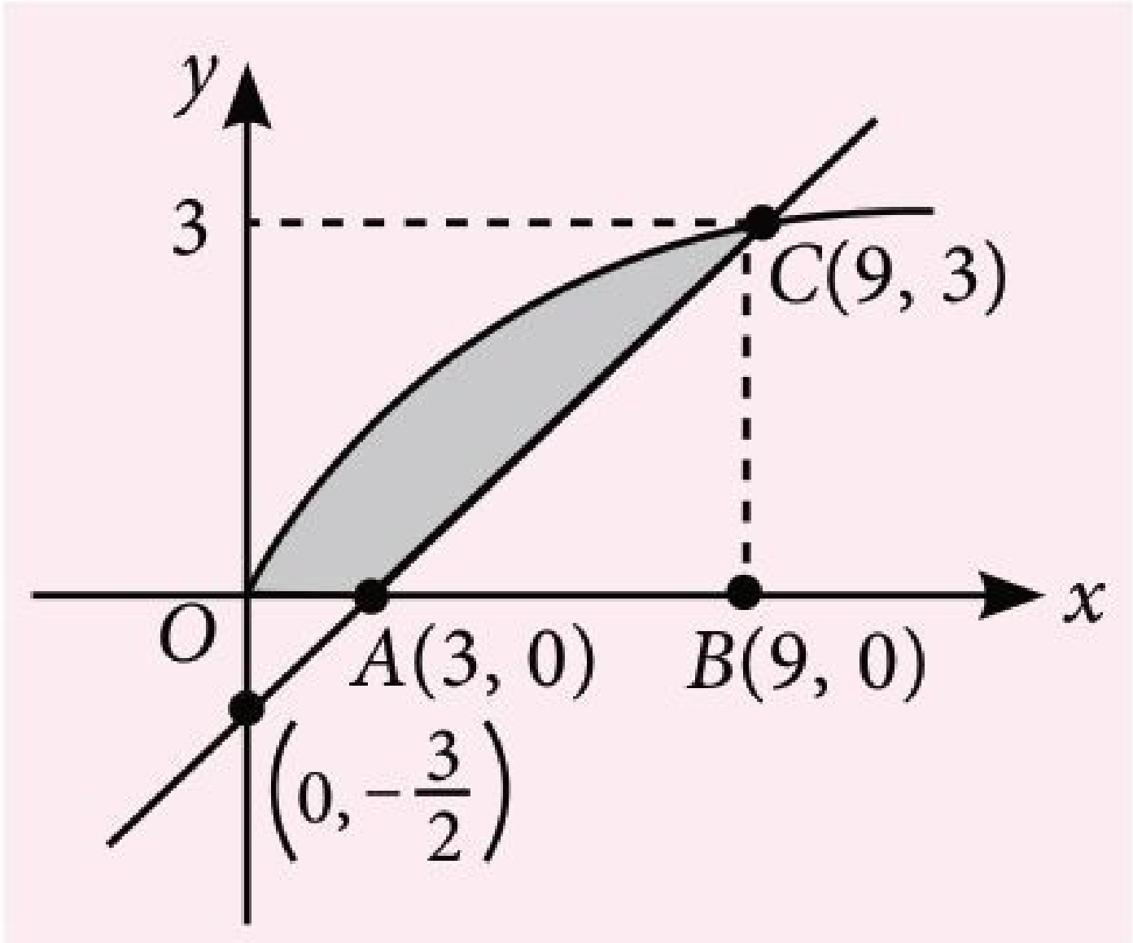
$y = \sqrt{x}$, $2y - x + 3 = 0$, $x$-axis

∴ Required area $= \int_0^9 \sqrt{x}dx - \text{Area of } \Delta ABC$

$$= \frac{2}{3}\left[x^{3/2}\right]_0^9 - \frac{1}{2}\times 6\times 3$$

$$= \frac{2}{3}\times(27-0) - 9$$

= 18 – 9 = 9 sq. units.



**6. (B) :** Let $P(t^2, t)$ be any point on the curve $x = y^2$. The distance $S$ of $P$ from the given line is

$$S = \left|\frac{t-t^2-1}{\sqrt{2}}\right| = \left|\frac{t^2-t+1}{\sqrt{2}}\right| = \frac{t^2-t+1}{\sqrt{2}}$$

[∵ $t^2 - t + 1 > 0$ for all $t \in R$]

∴ $\frac{dS}{dt} = \frac{2t-1}{\sqrt{2}}$

For the critical points of $S$, $\frac{dS}{dt} = 0$.

⇒ $2t - 1 = 0$ ⇒ $t = \frac{1}{2}$

Now, $\frac{d^2S}{dt^2} = \frac{2}{\sqrt{2}} = \sqrt{2} > 0$, for all $t$.

So, $S$ is minimum when $t = \frac{1}{2}$.

At $t = \frac{1}{2}$, the minimum value of $S = \frac{\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1}{\sqrt{2}} = \frac{3\sqrt{2}}{8}$.

**7. (B) :** We have, $z = x + y$

Subject to $x + y \le 3$, $-2x + y \le 1$, $x \le 2$, $x, y \ge 0$

Convert the given inequations into equations, we get

$x + y = 3$ ...(i)

$-2x + y = 1$ ...(ii)

$x = 2$ ...(iii)

Solving (i), (ii) and (iii), we get

Corner points as :

$A\left(\frac{2}{3}, \frac{7}{3}\right)$, $B(2, 1)$, $C(2, 0)$, $O(0, 0)$ and $D(0, 1)$

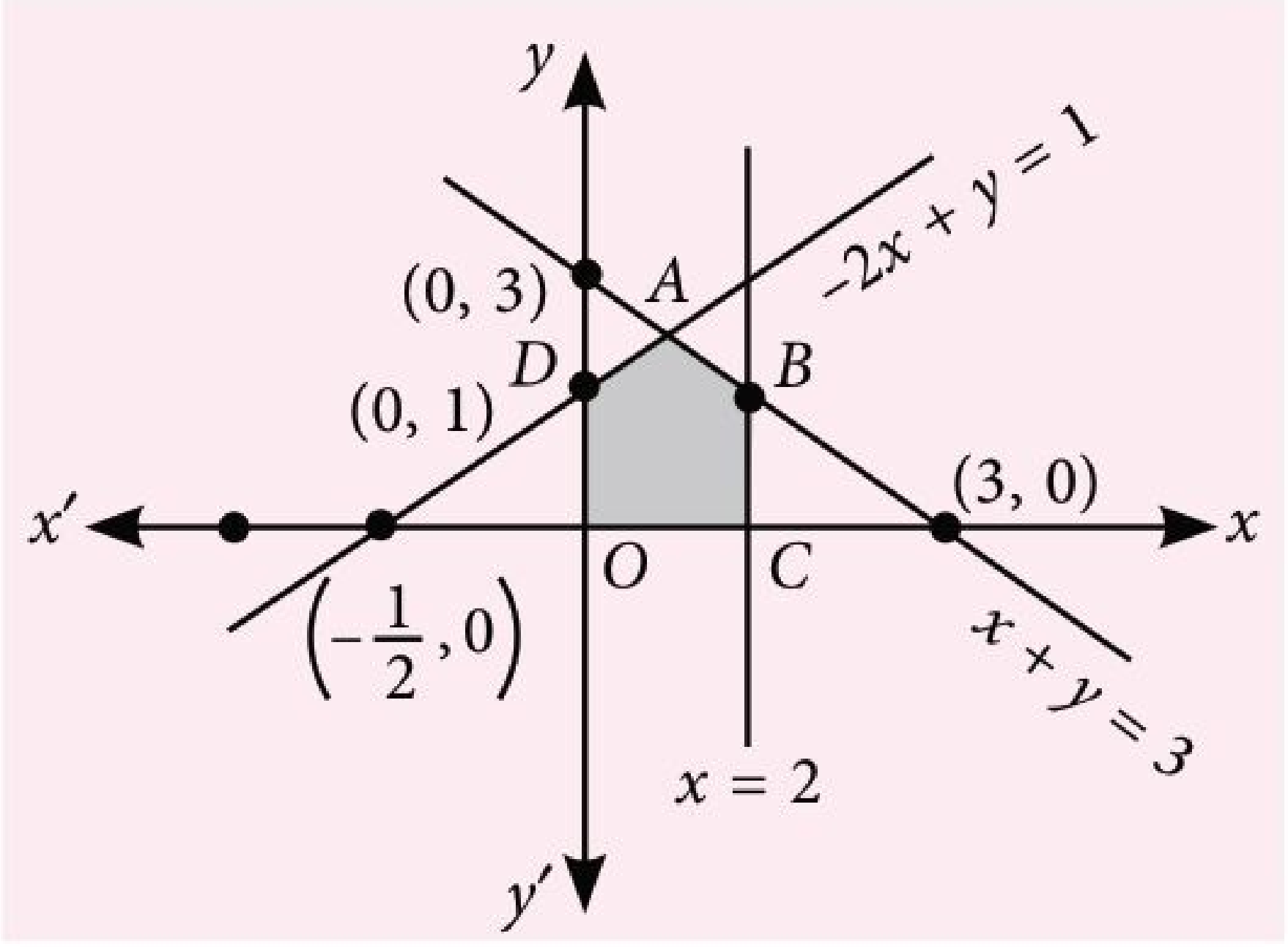
At $A\left(\frac{2}{3}, \frac{7}{3}\right)$, $z = 3$.

At $B(2, 1)$, $z = 3$,

At $(2, 0)$, $z = 2$, At $O(0, 0)$, $z = 0$,

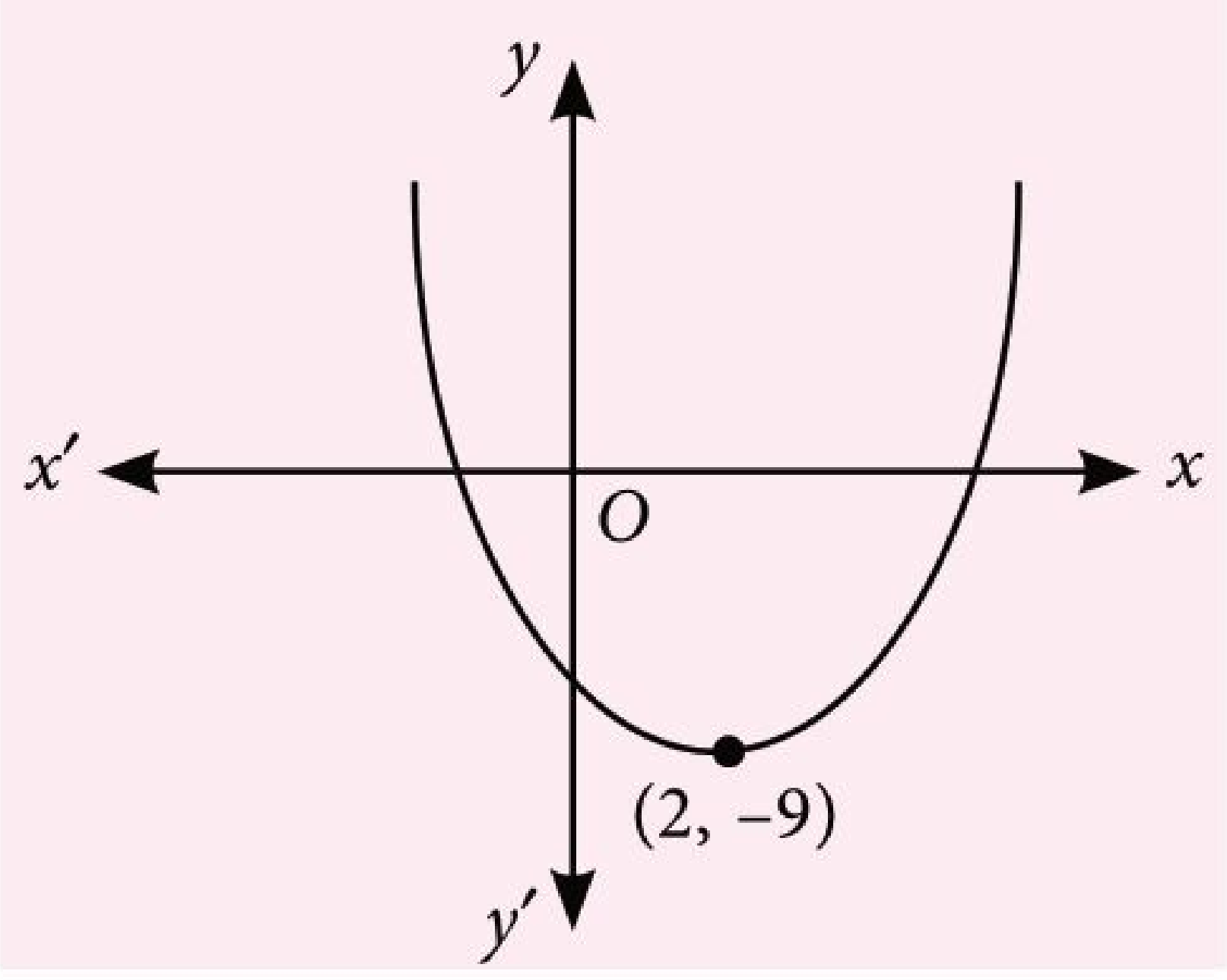
At $D(0, 1)$, $z = 1$

∴ The maximum value of $z$ is 3.



**8. (B) :** We have, $g(x) = x^2 - 4x - 5$

From the graph it is clear that g($x$) is one-one on (–∞, 2] or [2, ∞).

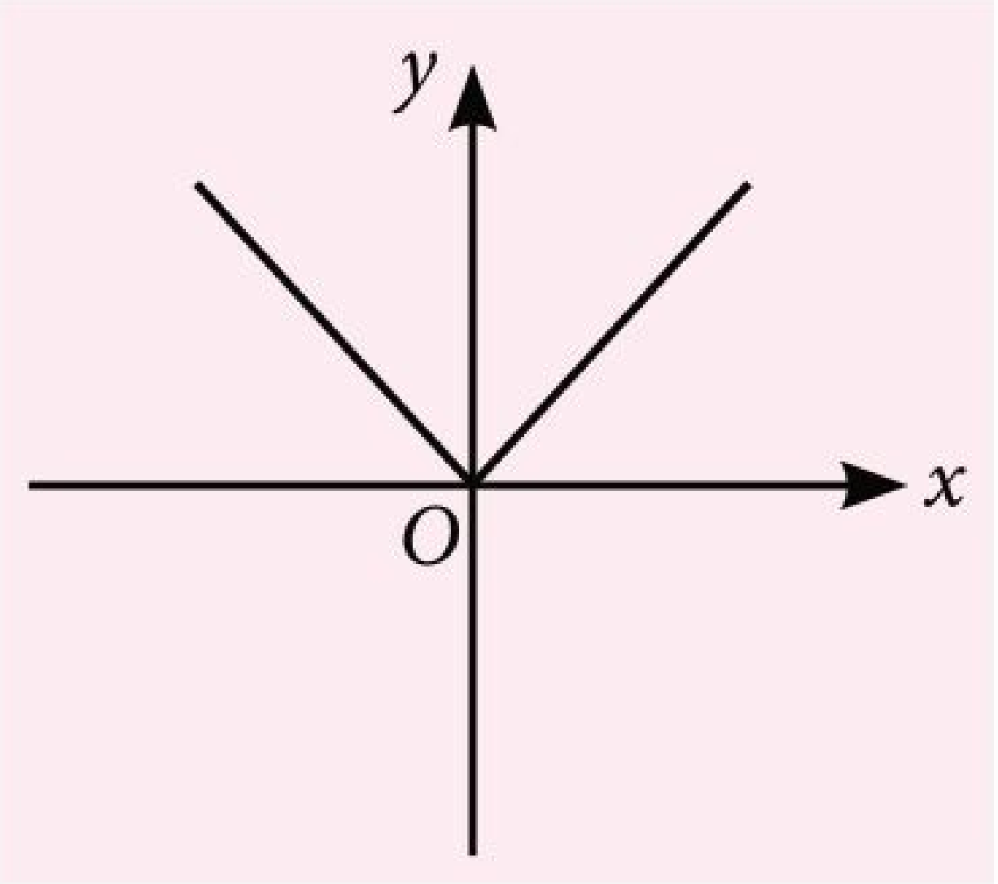


**9. (D) :** We have,

$f(x) = \sqrt[3]{x^2|x|}$

$f(x) = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

The function is differentiable on (–∞, 0) ∪ (0, ∞).



CONCEPT MAP

# PROBABILITY

Class XI

## Basic Terms

- **Experiment :** An operation which results in some well defined outcomes.
- **Random Experiment :** An experiment whose outcome cannot be predicted with certainty.
- **Sample space :** Set of all possible outcomes.
- **Sample point :** Elements of sample space.
- **Equally likely outcomes :** All outcomes with equal probability.

## Addition Theorem

- If $A$ and $B$ be any two events in a sample space $S$, then the probability of occurrence of atleast one of the events $A$ and $B$ is given by
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
  or in general for $n$ events $A_1, A_2, ..., A_n$, we have
  $$P(A_1 \cup A_2 \cup ... \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - ... + (-1)^{n-1} P(A_1 \cap A_2 \cap ... \cap A_n)$$
- If $A$ and $B$ are mutually exclusive events, then
  $P(A \cup B) = P(A) + P(B)$

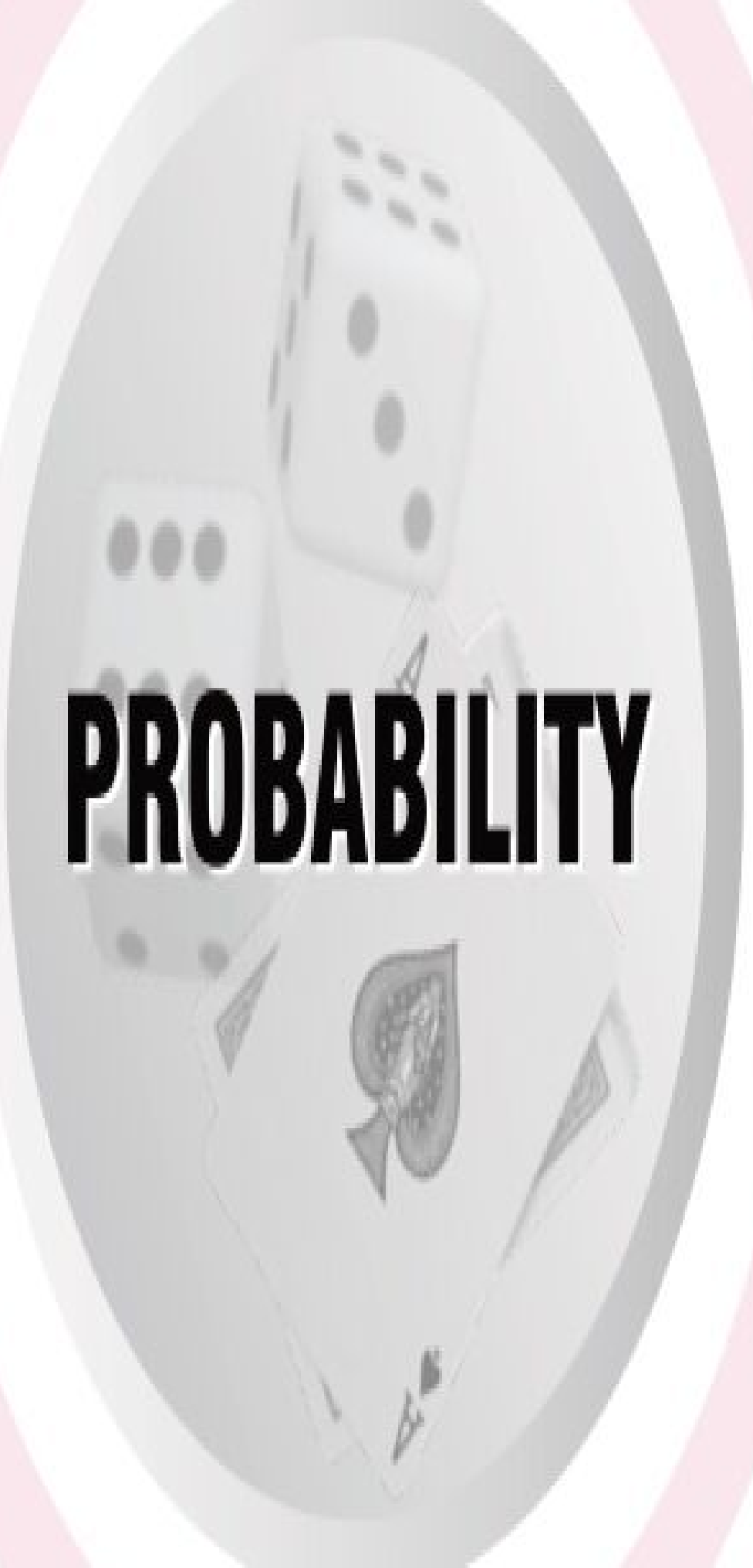
## Probability

Probability of an event$(E)$ is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of outcomes}}$$

In other words, number $P(x_i)$ associated with sample point $x_i$ in a sample space $S$, such that

- $0 \le P(x_i) \le 1$
- $\sum P(x_i) = 1 \ \forall \ x_i \in S$
- $P(A) = \sum P(x_i) \ \forall \ x_i \in A$. The number $P(x_i)$ is called probability of the outcome $x_i$.



## PROBABILITY

### Algebra of Events

Let S be sample space and $A$, $B$ be any set.

- **Complementary event :** $A' = S - A$
- **Event $A$ or $B$ :** Set $A \cup B$
- **Event $A$ and $B$ :** Set $A \cap B$
- **Event $A$ and not $B$ :** Set $A - B = A \cap B'$

## Events

It is a subset of a sample space.

## Some Important Results

Let A, B and C be any three events in a sample spaces, then

- $P$ (exactly one of the events occurs)
  $= P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A) + P(B) - 2P(A \cap B)$
- $P$ (atleast one of the events occurs)
  $= P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
- $P$ (atleast two of the events occur) $= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
- $P$ (exactly two of the events occur) $= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
- $P$(exactly one of the events occur) $= P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

### Types of Events

- **Impossible and Sure Events :** The empty set $\phi$ is called an impossible event and sample space $S$ is called the sure event.
- **Simple Event :** Event having only one sample point of a sample space.
- **Compound Event :** Event having more than one sample point.
- **Mutually Exclusive Events :** Set $A$ and $B$ are mutually exclusive if $A \cap B = \phi$
- **Exhaustive and Mutually Exclusive Events :** Events $E_1, E_2, ..., E_n$ are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup ... \cup E_n = S$ and $E_i \cap E_j = \phi \ \forall \ i \neq j$.

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# VECTOR ALGEBRA

Class XII

CONCEPT MAP

### Properties of Vector Product

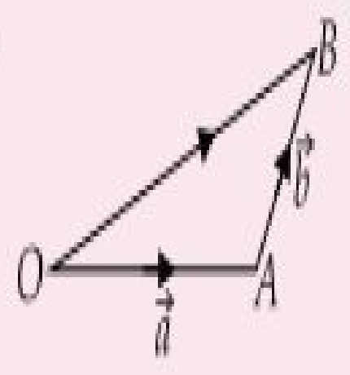
- $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$
- $\vec{a} \times \vec{a} = 0$
- $\vec{a} \times \vec{b} = 0 \Leftrightarrow \vec{a} \parallel \vec{b}$
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0, \hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$,
  $\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}, \hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then
  $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
- $(\alpha\vec{a}) \times (\beta\vec{b}) = (\alpha\beta)\vec{a} \times \vec{b}$
- $\vec{a} \times (\vec{b} \pm \vec{c}) = \vec{a} \times \vec{b} \pm \vec{a} \times \vec{c}$
- $(\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$
- If $\vec{a}$ and $\vec{b}$ are adjacent sides of a triangle, then area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$

### Vector (or cross) Product

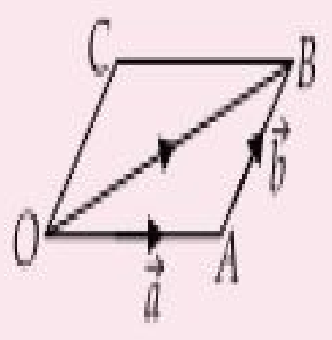
Vector product of vectors is given by $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|(\sin\theta)\hat{n}$, where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is the unit vector normal to the plane determined by $\vec{a}$ and $\vec{b}$.

## Addition of Two Vectors

- **Triangle Law :** In $\Delta OAB$, let position vectors of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ be $\vec{a}$ and $\vec{b}$ respectively. Then, $\overrightarrow{OB} = \vec{a} + \vec{b}$



- **Parallelogram Law :** In parallelogram $OABC$, let position vectors of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ be $\vec{a}$ and $\vec{b}$ respectively. Then, $\overrightarrow{OB} = \vec{a} + \vec{b}$



### Projection

Projection of a vector $\vec{a}$ on other vector $\vec{b}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

### Scalar (or Dot) Product

The scalar product of $\vec{a}$ and $\vec{b}$ written as $\vec{a} \cdot \vec{b}$ and is defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where $\theta$ is the angle between the vectors $\vec{a}$ and $\vec{b}$.

### Properties of Scalar Product

- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$
- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$,
- $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = \hat{j} \cdot \hat{i} = \hat{k} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$,
  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
  then, $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
- $\vec{a} \cdot (\alpha\vec{b}) = \alpha(\vec{a} \cdot \vec{b})$
- $\vec{a} \cdot (\vec{b} \pm \vec{c}) = \vec{a} \cdot \vec{b} \pm \vec{a} \cdot \vec{c}$

## Multiplication of Vectors

### Scalar Triple Product

Let $\vec{a}, \vec{b}$ are inclined at angle $\theta$ and $\vec{c}$ is inclined at angle $\phi$ with the vector $\vec{a} \times \vec{b}$.

$(\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{a}||\vec{b}|(\sin\theta)\hat{n} \cdot \vec{c}$
$= |\vec{a}||\vec{b}||\vec{c}| \sin\theta \cos\phi$

It is the volume of the parallelopiped with concurrent edges $\vec{a}, \vec{b}, \vec{c}$ and is denoted by the box product $[\vec{a}\,\vec{b}\,\vec{c}]$.

## VECTOR

A quantity that has magnitude as well as direction.

## Components of a Vector

Let $O$ be the origin and let $P(x, y, z)$ be any point in space then, $\overrightarrow{OP} = (x\hat{i} + y\hat{j} + z\hat{k})$, $x, y, z$ are scalar components of $\overrightarrow{OP}$ and $x\hat{i}, y\hat{j}, z\hat{k}$ are vector components of $\overrightarrow{OP}$.

Also, $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

### Coplanarity

Three vectors, $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar iff $\vec{a} \cdot (\vec{b} \times \vec{c})$ or $[\vec{a}\,\vec{b}\,\vec{c}] = 0$.

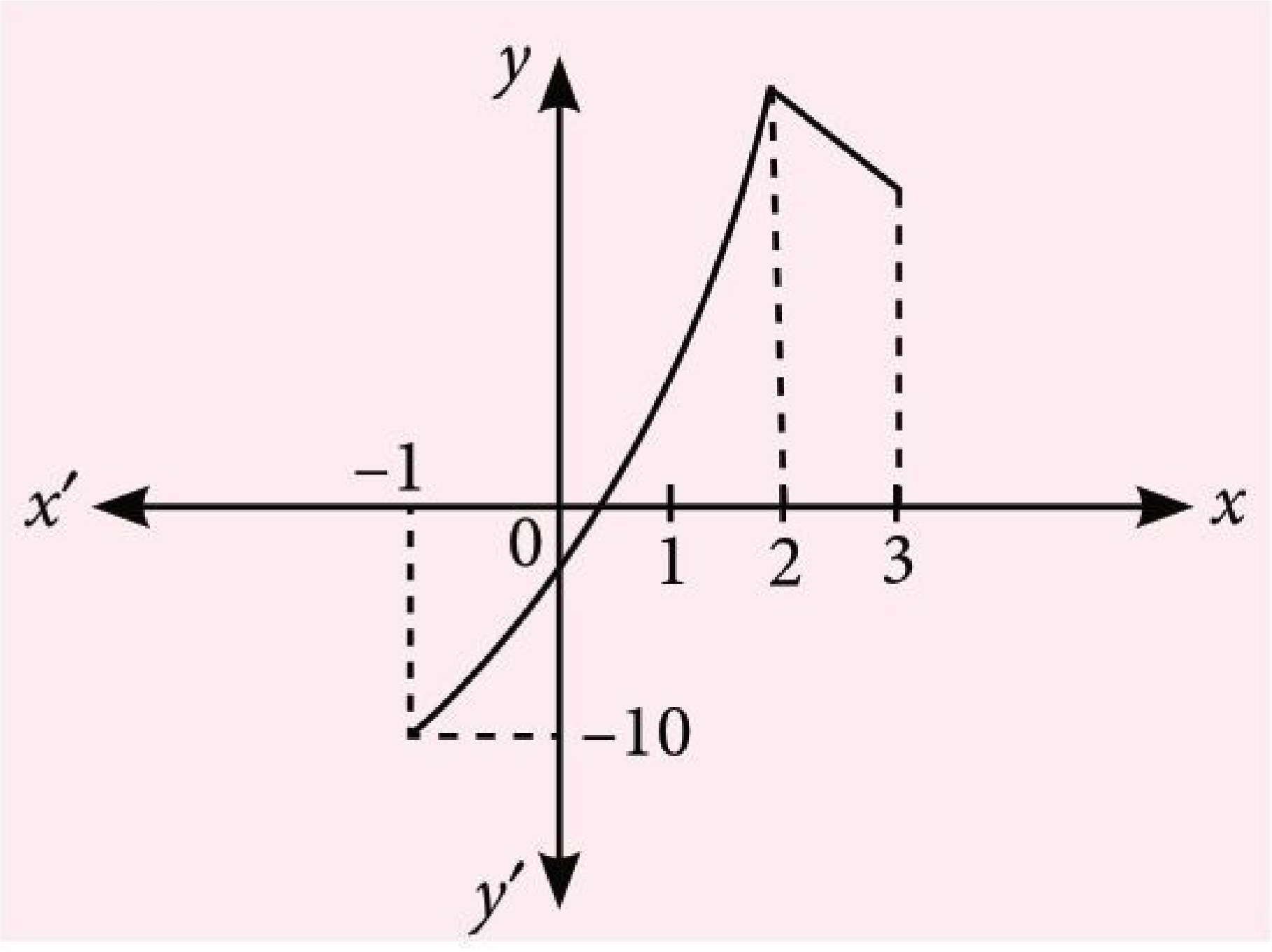
### Properties of Scalar Triple Product

- $[\alpha\vec{a}\ \beta\vec{b}\ \gamma\vec{c}] = \alpha\beta\gamma[\vec{a}\,\vec{b}\,\vec{c}]$
- $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\,\vec{b}\,\vec{c}]$
- $$[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$$
- $$[\vec{a}\,\vec{b}\,\vec{c}][\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} \vec{a} \cdot \vec{u} & \vec{a} \cdot \vec{v} & \vec{a} \cdot \vec{w} \\ \vec{b} \cdot \vec{u} & \vec{b} \cdot \vec{v} & \vec{b} \cdot \vec{w} \\ \vec{c} \cdot \vec{u} & \vec{c} \cdot \vec{v} & \vec{c} \cdot \vec{w} \end{vmatrix}$$
- $(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$
- $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$
- $[\vec{a}\,\vec{b}\,\vec{c}] = -[\vec{a}\,\vec{c}\,\vec{b}]$
- $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ if any two of the vectors are parallel
- $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ if $\vec{a}, \vec{b}, \vec{c}$ are coplanar or $\vec{c} = \alpha\vec{a} + \beta\vec{b}$
- $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$
  then $$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

**10. (D) :** $f(x)=\begin{cases}3x^2+12x-1, & -1\le x\le 2\\ 37-x, & -2<x\le 3\end{cases}$

From the graph clearly, $f(x)$ is increasing in $[-1, 2]$ and continuous in $[-1, 3]$.

Also, At $x = 2$, the function is not differentiable.



**11. (C) :** Let $y=f(x)=\sqrt{4-x^2}, 0\le x\le 2, 0\le y\le 2$

Let $x_1, x_2$ be any two element in the interval $0\le x\le 2$ such that $f(x_1) = f(x_2)$

$$\Rightarrow \sqrt{4-x_1^2}=\sqrt{4-x_2^2}=4-x_1^2=4-x_2^2$$

$$\Rightarrow x_1^2=x_2^2\Rightarrow x_1=x_2$$

[$\because x_1, x_2$ are non negative]

$\therefore$ $f(x)$ is one-one

Let $y \in [0, 2]$ be any arbitrary element, then there exists $x \in [0, 2]$ such that $f(x) = y \Rightarrow \sqrt{4-x^2}=y$

$\Rightarrow 4 - x^2 = y^2 \Rightarrow x^2 = 4 - y^2$

$\Rightarrow x=\pm\sqrt{4-y^2}\Rightarrow x=\sqrt{4-y^2}$ [$\because x$ is non negative]

Also, for $0 \le y \le 2$, we have

$0\le\sqrt{4-y^2}\le 2\Rightarrow 0\le x\le 2$, which is true.

Thus, for each $y\in[0, 2], \exists\ x=\sqrt{4-y^2}\in[0, 2]$

such that $f(x) = y$ $\therefore$ $f(x)$ is onto

Hence, $f$ is bijective function.

**12. (B) :** Let $\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}=x$

$$\Rightarrow \cos^{-1}\frac{\sqrt{5}}{3}=2x,\ 0\le 2x\le\pi$$

$$\Rightarrow \cos 2x=\frac{\sqrt{5}}{3}, 0\le x\le\frac{\pi}{2}\ \Rightarrow\ \frac{1-\tan^2 x}{1+\tan^2 x}=\frac{\sqrt{5}}{3}$$

$$\Rightarrow \sqrt{5}+\sqrt{5}\ \tan^2 x = 3 - 3\tan^2 x$$

$$\Rightarrow (3+\sqrt{5})\tan^2 x = 3-\sqrt{5}\Rightarrow \tan^2 x=\frac{3-\sqrt{5}}{3+\sqrt{5}}$$

$$\Rightarrow \tan x=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}\qquad \left(\because \text{ For } 0\le x\le\frac{\pi}{2}, \tan x\ge 0\right)$$

$$\Rightarrow \tan x=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}}=\frac{3-\sqrt{5}}{2}$$

$$\Rightarrow \tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)=\frac{3-\sqrt{5}}{2}.$$

**13. (C) :** Given $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

Also, $A + A' = \sqrt{2}I_2$

$$\Rightarrow \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}'=\sqrt{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{bmatrix}$$

Equating the corresponding elements, we get

$$2\cos\alpha=\sqrt{2}\Rightarrow\cos\alpha=\frac{1}{\sqrt{2}}\Rightarrow\alpha=\frac{\pi}{4}.$$

**14. (A) :** Since, the given points are collinear.

$$\therefore \begin{vmatrix}p+1 & 1 & 1\\ 2p+1 & 3 & 1\\ 2p+2 & 2p & 1\end{vmatrix}=0$$

$$\Rightarrow (p+1)(3-2p)-1(2p+1-2p-2)+1(4p^2+2p-6p-6)=0$$

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$\Rightarrow\ 3p - 2p^2 + 3 - 2p + 1 + 4p^2 - 4p - 6 = 0$

$\Rightarrow\ 2p^2 - 3p - 2 = 0\ \Rightarrow\ 2p^2 - 4p + p - 2 = 0$

$\Rightarrow\ 2p(p - 2) + 1(p - 2) = 0\ \Rightarrow\ p = 2, \dfrac{-1}{2}$

**15. (C) :** $f(x)=\dfrac{x}{\log x}\ \Rightarrow\ f'(x)=\dfrac{\log x-1}{(\log x)^2}$

For $f(x)$ to be increasing, we must have

$f'(x) > 0\ \Rightarrow\ \dfrac{\log x-1}{(\log x)^2}>0$

$\Rightarrow\ \log x - 1 > 0\qquad [\because (\log x)^2 > 0 \text{ for } x > 0, x \neq 1]$

$\Rightarrow\ \log x > 1$

$\Rightarrow\ x > e^1$

$[\because \log_a x > N \Rightarrow x > a^N$ for $a > 1$. Here, $e > 1\ \therefore\ \log_e x > 1$ $\Rightarrow x > e^1]$

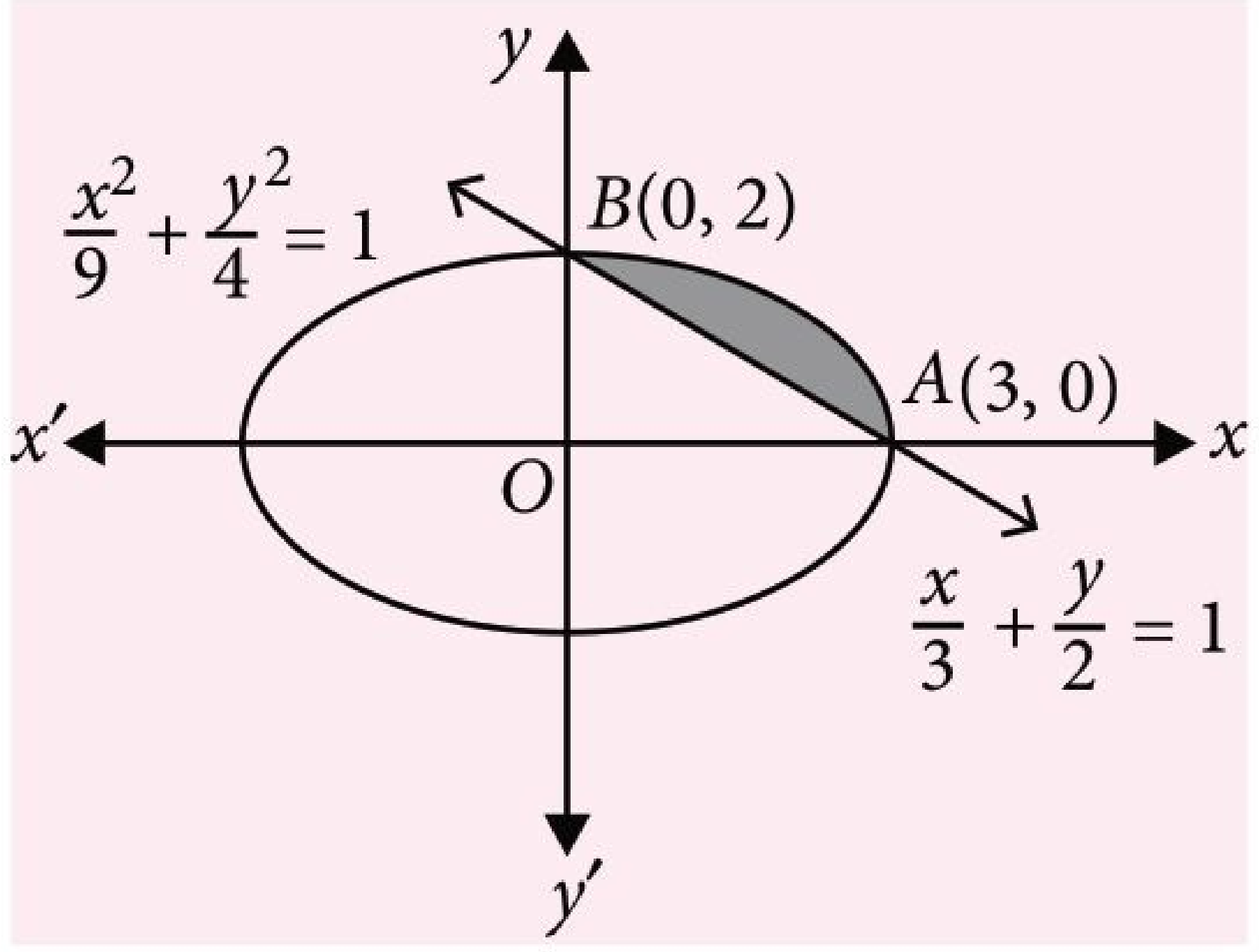
$\Rightarrow\ x \in (e, \infty)$.

So, $f(x)$ is increasing on $(e, \infty)$.

**16. (A) :** (i) : Given curve is $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$, represents an ellipse with centre at (0, 0).

and equation of line is $\dfrac{x}{3}+\dfrac{y}{2}=1$

Thus, intersection points are $A(3, 0)$ and $B(0, 2)$.



$\therefore$ Required area = Area of shaded region

$$=\int_0^3 2\sqrt{1-\frac{x^2}{9}}\,dx-\int_0^3 2\left(1-\frac{x}{3}\right)dx$$

$$=\frac{2}{3}\int_0^3\sqrt{3^2-x^2}\,dx-\frac{2}{3}\int_0^3(3-x)\,dx$$

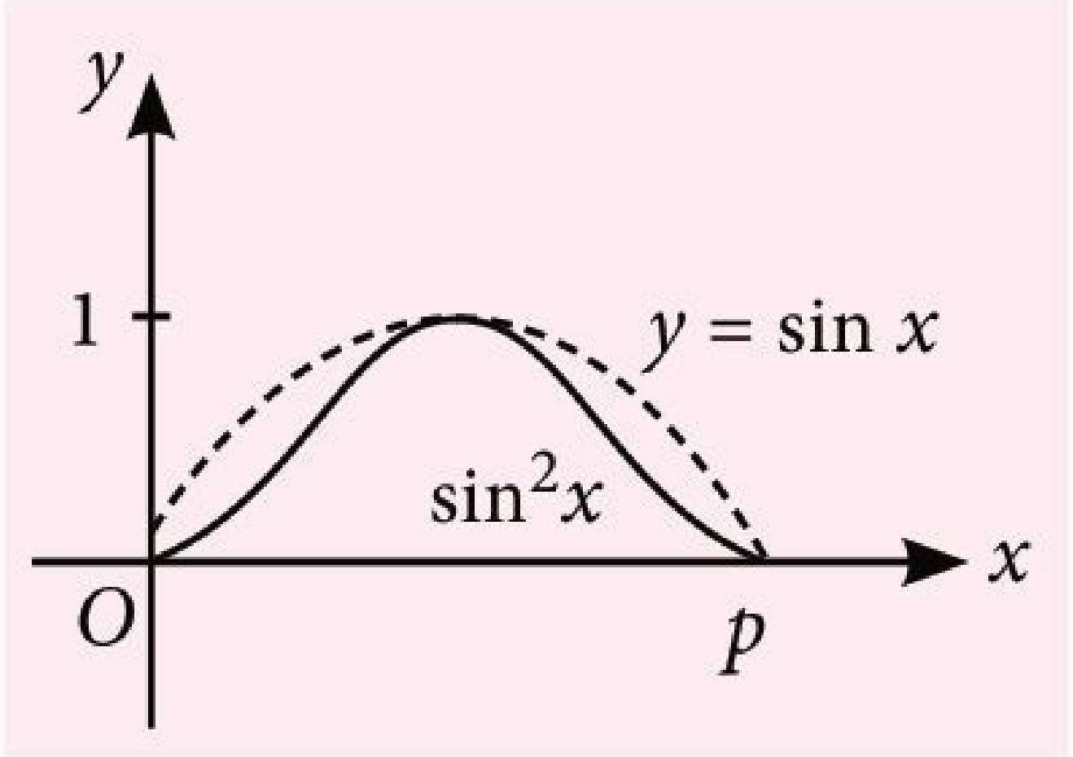
$$=\frac{2}{3}\left[\frac{x}{2}\sqrt{3^2-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3-\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

$$=\frac{2}{3}\left[0+\frac{9}{2}\sin^{-1}(1)-0\right]-\frac{2}{3}\left[9-\frac{9}{2}-0\right]$$

$$=3\left(\frac{\pi}{2}\right)-(3)=3\left(\frac{\pi}{2}-1\right)=\frac{3}{2}(\pi-2)\text{ sq. units}$$

(ii) : For $0 < t < 1$, $t^2 < t$

$\therefore\ \sin^2 x < \sin x$



$\Rightarrow\ \int_0^\pi \sin^2 x\,dx < \int_0^\pi \sin x\,dx$

(iii): $\int e^x\{f(x)+f'(x)\}dx=\int e^x f(x)dx+\int e^x f'(x)dx$

$=f(x)\cdot e^x-\int f'(x)e^x dx+\int e^x f'(x)dx\ = e^x f(x) + C$

For $f(x) = \tan x$, $f'(x) = \sec^2 x$

$\therefore\ \int e^x(\tan x+\sec^2 x)dx=e^x\cdot\tan x+C$

**17. (C) :** (i) We have, $2\dfrac{d^2y}{dx^2}+3\sqrt{1-\left(\dfrac{dy}{dx}\right)^2}-y=0$

$\therefore\ 2\dfrac{d^2y}{dx^2}=-3\sqrt{1-\left(\dfrac{dy}{dx}\right)^2}-y$ ...(1)

Squaring (1) on both sides, we get

$$4\left(\frac{d^2y}{dx^2}\right)^2=9\left[1-\left(\frac{dy}{dx}\right)^2-y\right]$$

Now, highest order derivative is $\dfrac{d^2y}{dx^2}$.

So, its degree is 2.

(ii) : We have, $\dfrac{dy}{dx}=(4x+y+1)^2$ ...(1)

Put $4x + y + 1 = z\ \Rightarrow\ 4+\dfrac{dy}{dx}=\dfrac{dz}{dx}\ \Rightarrow\ \dfrac{dy}{dx}=\dfrac{dz}{dx}-4$

$\therefore$ From (1), we have $\dfrac{dz}{dx}-4=z^2\ \Rightarrow\ \dfrac{dz}{z^2+4}=dx$

Integrating both sides w.r.t. $x$, we get

$\int\dfrac{dz}{z^2+4}=\int dx+C\ \Rightarrow\ \dfrac{1}{2}\tan^{-1}\left(\dfrac{z}{2}\right)=x+C$

$\Rightarrow\ \dfrac{1}{2}\tan^{-1}\left(\dfrac{4x+y+1}{2}\right)=x+C$

(iii) We have, $\dfrac{dy}{dx}=e^{x-y}(e^x-e^y)$

## MONTHLY TEST DRIVE CLASS XII ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (d) | **2.** | (d) | **3.** | (b) | **4.** | (d) | **5.** | (c) |
| **6.** | (d) | **7.** | (a,c) | **8.** | (b,d) | **9.** | (a, c) | **10.** | (a,b,c,d) |
| **11.** | (a,b,c) | **12.** | (a,b,c) | **13.** | (b) | **14.** | (d) | **15.** | (a) |
| **16.** | (c) | **17.** | (64) | **18.** | (2) | **19.** | (1) | **20.** | (4) |

$$\Rightarrow \frac{dy}{dx} = \frac{e^{2x}}{e^y} - e^x \Rightarrow e^y\frac{dy}{dx} + e^x e^y = e^{2x} \qquad ...(1)$$

Put $e^y = v \Rightarrow e^y\frac{dy}{dx} = \frac{dv}{dx}$

Thus, (1) reduces to $\frac{dv}{dx} + e^x \cdot v = e^{2x}$

$\text{I.F.} = e^{\int e^x dx} = e^{e^x}$

Thus, solution is $v \cdot e^{e^x} = \int e^{e^x} \cdot e^x \cdot e^x dx + c$

Putting $e^x = t \Rightarrow e^x\, dx = dt$

$v \cdot e^{e^x} = \int e^t t\, dt + C \Rightarrow v \cdot e^{e^x} = e^t(t-1) + C$

$\Rightarrow e^y \cdot e^{e^x} = e^{e^x}(e^x - 1) + c$

$\Rightarrow e^y = e^x - 1 + ce^{-e^x}$

**18. (A) :** Since, $P(A) = P(A|B) = \frac{1}{4}$

$\Rightarrow$ Events $A$ & $B$ are independent.

$\therefore\ P(B) = P(B|A) = \frac{1}{2}$

Now, $P(A|B) = \frac{1}{4}$ & $P(B|A) = \frac{1}{2}$

$\therefore\ P(\bar{A}|B) = 1 - P(A|B) = 1 - \frac{1}{4} = \frac{3}{4}$

and $P(\bar{B}|A) = 1 - P(B|A) = 1 - \frac{1}{2} = \frac{1}{2}$.

Hence, both Statement-I and Statement-II are true.

**19. (B) : (P)** We have, $|\vec{a} + \vec{b}| = |\vec{a} + 2\vec{b}|$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a}\cdot\vec{b}$

$\Rightarrow 2\vec{a}\cdot\vec{b} = -3|\vec{b}|^2 < 0$

Hence, angle between $\vec{a}$ and $\vec{b}$ is obtuse.

**(Q)** We have, $|\vec{a} + \vec{b}| = |\vec{a} - 2\vec{b}|$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + 4|\vec{b}|^2 - 4\vec{a}\cdot\vec{b}$

$\Rightarrow 6\vec{a}\cdot\vec{b} = 3|\vec{b}|^2 > 0$

Hence, angle between $\vec{a}$ and $\vec{b}$ is acute.

**(R)** We have, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a}$ is perpendicular to $\vec{b}$.

**(S)** $\vec{c} \times (\vec{a} \times \vec{b})$ lies in the plane of vector $\vec{a}$ and $\vec{b}$. A vector perpendicular to this plane is parallel to $\vec{a} \times \vec{b}$. Hence, angle is 0°.

**20. (C) :** Direction ratios of $RS$ are

$0 - 6, 2 - 4, 1 - 4$ or $-6, -2, -3$

$\therefore$ Its direction cosines $(l, m, n)$ are $\frac{-6}{7}, \frac{-2}{7}, \frac{-3}{7}$

$\therefore$ Projection of $PQ$ on $RS$

$= l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

$= -\frac{6}{7}(2-3) - \frac{2}{7}(5-6) - \frac{3}{7}(2-4)$

$= \left(-\frac{6}{7}\right)(-1) + \left(\frac{-2}{7}\right)(-1) + \left(\frac{-3}{7}\right)(-2) = \frac{6}{7} + \frac{2}{7} + \frac{6}{7} = 2$

*For other sections/subjects please refer to Physics For You, Chemistry Today and Biology Today*

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Mentor Advice

## 5 Steps to Excel in Olympiads

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CLASS 11 CLASS 12

1. **Make an Appropriate Use of Time Tool :** Get to know your syllabus, make a timetable for various subjects, giving more time to the subjects you are weak in.
2. **Keep Revising at Regular Intervals :** Revise all the topics at regular intervals, as it is imperative for any exam preparation. It helps in remembering the facts, figures, topics and methodologies already covered by you and will surely increase your confidence.
3. **Practice Previous Years' Papers :** Practice as many previous years' question papers as possible, as it gives a fair idea about the level of difficulty and pattern of questions.
4. **Study with Good Resource Books :** A good resource book is like a good 'Mentor'. Prepare for Olympiads with right guidance as well as with the best trusted resources.
5. **Stay Motivated :** Last but not the least, always stay positive and focused. Believe in Yourself!

# YOU ASK
# WE ANSWER

**Do you have a question that you just can't get answered?**
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

**1.** Solve the differential equation :

$$\frac{dy}{dx}+\frac{(ax+hy+g)}{(hx+by+f)}=0$$

*(Soham Bose, West Bengal)*

**Ans.** Given differential equation is $\frac{dy}{dx}+\frac{ax+hy+g}{hx+by+f}=0$

$\Rightarrow\quad (hx+by+f)dy+(ax+hy+g)dx=0$

$\Rightarrow\quad hx\,dy+by\,dy+f\,dy+ax\,dx+hy\,dx+gdx=0$

$\Rightarrow\quad h(x\,dy+y\,dx)+by\,dy+f\,dy+ax\,dx+gdx=0$

$\Rightarrow\quad h\,d(xy)+by\,dy+f\,dy+ax\,dx+gdx=0$

On integrating both sides, we get

$hxy+b\frac{y^2}{2}+fy+a\frac{x^2}{2}+gx=c$, which is the required solution.

**2.** The sum of squares of the perpendicular drawn from the points (0, 1) and (0, –1) to any tangent to a curve is 2. Find the equation of the curve.

*(Parul chouhan, Delhi)*

**Ans.** The equation of any tangent to a curve $y=f(x)$ is

$$y-y_1=\frac{dy}{dx}(x-x_1)\ \Rightarrow\ x\frac{dy}{dx}-y+\left(y_1-x_1\frac{dy}{dx}\right)=0$$

According to the problem, we have

$$\left|\frac{-1+y_1-x_1\frac{dy}{dx}}{\sqrt{\left(\frac{dy}{dx}\right)^2+1}}\right|^2+\left|\frac{1+y_1-x_1\frac{dy}{dx}}{\sqrt{\left(\frac{dy}{dx}\right)^2+1}}\right|^2=2$$

$$\Rightarrow\ \left\{\left(y_1-x_1\frac{dy}{dx}\right)-1\right\}^2+\left\{\left(y_1-x_1\frac{dy}{dx}\right)+1\right\}^2=2\left\{\left(\frac{dy}{dx}\right)^2+1\right\}$$

$$\Rightarrow\ 2\left\{\left(y_1-x_1\frac{dy}{dx}\right)^2+1\right\}=2\left\{\left(\frac{dy}{dx}\right)^2+1\right\}$$

$$[\because\ (a-b)^2+(a+b)^2=2(a^2+b^2)]$$

$$\Rightarrow\ \left(y_1-x_1\frac{dy}{dx}\right)^2=\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow\ y-x\frac{dy}{dx}=\pm\frac{dy}{dx}\quad [\text{writing } x_1 \text{ as } x \text{ and } y_1 \text{ as } y]$$

$$\Rightarrow\ \frac{1}{x\pm1}dx=\frac{1}{y}dy\quad\Rightarrow\quad \log(x\pm1)=\log y+\log K$$

$$\Rightarrow\ Ky=x\pm1\ \Rightarrow\ y=C(x\pm1)$$

**3.** Find the roots of the equation, $(x^2+1)^2=x(4x^2+5x+4)$.

*(Vinay Paul, Kerala)*

**Ans.** Given equation is

$$(x^2+1)^2=x(4x^2+5x+4)\qquad\text{...(i)}$$

$$\Rightarrow\ x^4-4x^3-3x^2-4x+1=0$$

$$\Rightarrow\ x^2\left(x^2-4x-3-\frac{4}{x}+\frac{1}{x^2}\right)=0$$

$$\Rightarrow\ \left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)-3=0$$

(as $x=0$ does not satisfy (i))

$$\Rightarrow\ t^2-4t-5=0,\ \text{where } t=x+\frac{1}{x}\ \Rightarrow (t+1)(t-5)=0$$

$$\Rightarrow\ \left(x+\frac{1}{x}+1\right)\left(x+\frac{1}{x}-5\right)=0$$

$$\Rightarrow\ (x^2+x+1)(x^2-5x+1)=0$$

$$\Rightarrow\ x^2+x+1=0,\ x^2-5x+1=0$$

$$\Rightarrow\ x=\frac{-1\pm i\sqrt{3}}{2},\ x=\frac{5\pm\sqrt{21}}{2}$$

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## SAMURAI SUDOKU

### ANSWER - OCTOBER 2022

| | | | | | | | | | | | | | | | | | | | | |
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| 1 | 3 | 8 | 6 | 9 | 4 | 2 | 7 | 5 | | | | 1 | 3 | 8 | 9 | 5 | 4 | 2 | 6 | 7 |
| 2 | 8 | 5 | 1 | 6 | 3 | 4 | 9 | 7 | | | | 5 | 7 | 4 | 1 | 6 | 8 | 9 | 2 | 3 |
| 4 | 9 | 1 | 7 | 5 | 2 | 8 | 6 | 3 | | | | 9 | 8 | 2 | 3 | 7 | 5 | 1 | 4 | 6 |
| 3 | 7 | 6 | 9 | 4 | 8 | 5 | 2 | 1 | | | | 3 | 6 | 1 | 4 | 9 | 2 | 7 | 5 | 8 |
| 8 | 4 | 7 | 2 | 3 | 6 | 1 | 5 | 9 | 7 | 4 | 8 | 6 | 2 | 3 | 5 | 1 | 9 | 8 | 7 | 4 |
| 9 | 1 | 2 | 4 | 7 | 5 | 3 | 8 | 6 | 2 | 5 | 1 | 7 | 4 | 9 | 2 | 8 | 6 | 3 | 1 | 5 |
| 6 | 5 | 3 | 8 | 1 | 9 | 7 | 4 | 2 | 3 | 6 | 9 | 8 | 1 | 5 | 7 | 4 | 3 | 6 | 9 | 2 |
| | | | | | | 4 | 3 | 8 | 5 | 2 | 7 | 9 | 6 | 1 | | | | | | |
| | | | | | | 9 | 1 | 7 | 6 | 8 | 3 | 4 | 5 | 2 | | | | | | |
| | | | | | | 2 | 6 | 5 | 1 | 9 | 4 | 3 | 7 | 8 | | | | | | |
| 7 | 6 | 8 | 4 | 3 | 2 | 5 | 9 | 1 | 4 | 3 | 6 | 2 | 8 | 7 | 9 | 6 | 3 | 5 | 4 | 1 |
| 5 | 3 | 9 | 6 | 1 | 7 | 8 | 2 | 4 | 9 | 7 | 5 | 1 | 3 | 6 | 5 | 7 | 4 | 9 | 8 | 2 |
| 4 | 1 | 2 | 5 | 8 | 9 | 6 | 7 | 3 | 8 | 1 | 2 | 5 | 9 | 4 | 8 | 1 | 2 | 6 | 3 | 7 |
| 8 | 7 | 4 | 3 | 2 | 6 | 1 | 5 | 9 | | | | 4 | 2 | 5 | 6 | 8 | 9 | 1 | 7 | 3 |
| 6 | 2 | 1 | 8 | 9 | 5 | 3 | 4 | 7 | | | | 7 | 1 | 3 | 2 | 4 | 5 | 8 | 9 | 6 |
| 9 | 5 | 3 | 1 | 7 | 4 | 2 | 8 | 6 | | | | 9 | 6 | 8 | 1 | 3 | 7 | 2 | 5 | 4 |
| 2 | 4 | 6 | 9 | 5 | 1 | 7 | 3 | 8 | | | | 8 | 4 | 9 | 3 | 2 | 1 | 7 | 6 | 5 |
| 1 | 8 | 7 | 2 | 4 | 3 | 9 | 6 | 5 | | | | 3 | 5 | 2 | 7 | 9 | 6 | 4 | 1 | 8 |
| 3 | 9 | 5 | 7 | 6 | 8 | 4 | 1 | 2 | | | | 6 | 7 | 1 | 4 | 5 | 8 | 3 | 2 | 9 |

# beat the TIME TRAP

**Duration : 60 minutes**

## SECTION-I

### Single Option Correct Type

**1.** The equation of the circle passing through the focii of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at (0, 3) is

(a) $x^2+y^2-6y+7=0$ (b) $x^2+y^2-6y-5=0$

(c) $x^2+y^2-6y+5=0$ (d) $x^2+y^2-6y-7=0$

**2.** If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors equally inclined to each other at an angle $\alpha$ ($\neq 0$) then the angle between $\vec{a}\times\vec{b}$ and the plane containing $\vec{b}$ and $\vec{c}$ is

(a) $\sin^{-1}\left(\tan\frac{\alpha}{2}\cdot|\cot\alpha|\right)$ (b) $\cos^{-1}\left(\tan\frac{\alpha}{2}\cdot|\cot\alpha|\right)$

(c) $\cos^{-1}\left(\cot\frac{\alpha}{2}\cdot|\tan\alpha|\right)$ (d) $\sin^{-1}\left(\cot\frac{\alpha}{2}\cdot|\tan\alpha|\right)$

**3.** The value of '$a$' for which the system of equations $(a+1)^3x+(a+2)^3y=(a+3)^3$, $(a+1)x+(a+2)y=a+3$ and $x+y=1$ is consistent, is

(a) $-2$ (b) $-1$ (c) 1 (d) 2

**4.** Let $AD$ be the angular bisector of the angle $A$ of $\Delta ABC$, then $\overrightarrow{AD}=\alpha\overrightarrow{AB}+\beta\overrightarrow{AC}$, where

(a) $\alpha=\frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}+\overrightarrow{AC}|}$, $\beta=\frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}+\overrightarrow{AC}|}$

(b) $\alpha=\frac{|\overrightarrow{AB}|+|\overrightarrow{AC}|}{|\overrightarrow{AB}|}$, $\beta=\frac{|\overrightarrow{AB}|+|\overrightarrow{AC}|}{|\overrightarrow{AC}|}$

(c) $\alpha=\frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$, $\beta=\frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$

(d) $\alpha=\left|\frac{\overrightarrow{AB}}{\overrightarrow{AC}}\right|$, $\beta=\left|\frac{\overrightarrow{AC}}{\overrightarrow{AB}}\right|$

**5.** If $\int f(x)dx=\psi(x)$ then $\int x^5 f(x^3)\,dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(b) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

(c) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

(d) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$

**6.** The number of solutions of the equation $16(\sin^5x+\cos^5x)=11(\sin x+\cos x)$ in the interval $[0,2\pi]$ is

(a) 6 (b) 7 (c) 8 (d) 9

**7.** If $\cos\theta=\frac{a\cos\phi+b}{a+b\cos\phi}$, then $\tan\frac{\theta}{2}$ is equal to

(a) $\sqrt{\left(\frac{a-b}{a+b}\right)}\tan(\phi/2)$ (b) $\sqrt{\left(\frac{a+b}{a-b}\right)}\cos(\phi/2)$

(c) $\sqrt{\left(\frac{a-b}{a+b}\right)}\sin(\phi/2)$ (d) none of these

**8.** Let $x>0$, $y>0$, $z>0$ are respectively the 2nd, 3rd, 4th terms of a G.P and

$$\Delta=\begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2} \end{vmatrix}=(r-1)^2\left(1-\frac{1}{r^2}\right)$$

(where $r$ is the common ratio), then

(a) $k=-1$ (b) $k=1$

(c) $k=0$ (d) None of these

**9.** Domain of the function $f(x)=\sqrt{5|x|-x^2-6}$ is

(a) $(-\infty,2)\cup(3,\infty)$ (b) $[-3,-2]\cup[2,3]$

(c) $(-\infty,-2)\cup(2,3)$ (d) $R-\{-3,-2,2,3\}$

**10.** The period of the function $f(x)=(-1)^{[x]}$ where $[\cdot]$ = G.I.F

(a) 2 (b) 1 (c) 3 (d) 4

## SECTION-II

### Numerical Answer Type

**11.** If $\sin\left\{2\left(\sin^{-1}\frac{\sqrt{5}}{3}-\cos^{-1}\frac{\sqrt{5}}{3}\right)\right\}$ is equal to $\frac{k\sqrt{5}}{81}$, then $k =$ ________.

**12.** If $\int(x^9+x^6+x^3)(2x^6+3x^3+6)^{1/3}dx$

$=\frac{1}{A}(2x^9+3x^6+6x^3)^B+K$, then the value of $\frac{AB}{4}$ is ____.

**13.** If the mean and the variance of a binomial variate $X$ are 2 and 1 respectively, then $16\ P(X \geq 1) =$_______.

**14.** Let $\vec{a}=-\hat{i}-\hat{k}, \vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a}=0$, then $\vec{r}\cdot\vec{b}=$ _______.

**15.** Water is dropped at the rate of 2 m$^3$/s into a cone of semi vertical angle 45°. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 m is $d$, then the value of $5\ d$ is _______.

## SOLUTIONS

**1. (d) :** Foci are given by $(\pm\ ae, 0)$

As $a^2e^2 = a^2 - b^2 = 7$ we have equation of circle as

$(x - 0)^2 + (y - 3)^2 = (\sqrt{7} - 0)^2 + (0 - 3)^2$

$\therefore \quad x^2 + y^2 - 6y - 7 = 0$

**2. (a) :** Let $\theta$ be the required angle, then

$$\sin\theta=\cos(90°-\theta)=\left|\frac{(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})}{|\vec{a}\times\vec{b}|\cdot|\vec{b}\times\vec{c}|}\right|$$

$$=\left|\frac{[(\vec{a}\times\vec{b})\times\vec{b}]\cdot\vec{c}}{\sin^2\alpha}\right|=\left|\frac{[(\vec{a}\cdot\vec{b})\vec{b}-(\vec{b}\cdot\vec{b})\vec{a}]\cdot\vec{c}}{\sin^2\alpha}\right|$$

$$=\left|\frac{\cos\alpha\vec{b}\cdot\vec{c}-\vec{a}\cdot\vec{c}}{\sin^2\alpha}\right|=\left|\frac{\cos^2\alpha-\cos\alpha}{\sin^2\alpha}\right|$$

$$=\left|-\frac{\cos\alpha(1-\cos\alpha)}{\sin\alpha\sin\alpha}\right|=\left|-\frac{\cos\alpha\cdot 2\sin^2(\alpha/2)}{\sin\alpha\cdot 2\sin(\alpha/2)\ \cos(\alpha/2)}\right|$$

$$=\left|-\cot\alpha\tan\frac{\alpha}{2}\right|=\left|\cot\alpha\tan\frac{\alpha}{2}\right|$$

$$\therefore \quad \theta=\sin^{-1}\left(\tan\frac{\alpha}{2}|\cot\alpha|\right)$$

**3. (a) :** The system of equations is consistent, if the determinant of coefficients is zero.

$$\Rightarrow \begin{vmatrix}(a+1)^3 & (a+2)^3 & (a+3)^3\\ a+1 & a+2 & a+3\\ 1 & 1 & 1\end{vmatrix}=0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_2$, we get

$$\begin{vmatrix}(a+1)^3 & 3a^2+9a+7 & 3a^2+15a+19\\ a+1 & 1 & 1\\ 1 & 0 & 0\end{vmatrix}=0$$

$\Rightarrow \quad (3a^2 + 9a + 7) - (3a^2 + 15a + 19) = 0$

$\Rightarrow \quad -6a - 12 = 0$ or $a = -2$

**4. (c) :** $\because \quad \frac{BD}{DC}=\left|\frac{\overrightarrow{AB}}{\overrightarrow{AC}}\right|; \qquad BD:DC=|\overrightarrow{AB}|:|\overrightarrow{AC}|$

$$\therefore \quad \overrightarrow{AD}=\frac{BD\overrightarrow{AC}+DC\overrightarrow{AB}}{(BD+DC)}=\frac{\frac{BD}{DC}\overrightarrow{AC}+\overrightarrow{AB}}{\left(\frac{BD}{DC}+1\right)}$$

$$=\frac{\overrightarrow{AB}|\overrightarrow{AC}|+\overrightarrow{AC}|\overrightarrow{AB}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$$

## SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 8 | | | | | | 1 | 9 | | | | 2 | 7 | | | | | | 1 | 5 |
| 4 | | 6 | | | | 2 | | 5 | | | | 3 | | 6 | | | | 4 | | 9 |
| | 7 | 1 | | | | 4 | 6 | | | | | | 1 | 9 | | | | 3 | 2 | |
| | | | 1 | | 6 | | | | | | | | | | 5 | | 4 | | | |
| | | | | 5 | | | | | | | | | | | | 9 | | | | |
| | | | 9 | | 4 | | | | | | | | | | 8 | | 6 | | | |
| | 1 | 3 | | | | 6 | 5 | | | | | | 9 | 7 | | | | 6 | 4 | |
| 8 | | 9 | | | | 1 | | 4 | | | | 8 | | 5 | | | | 7 | | 2 |
| 6 | 2 | | | | | | 8 | 3 | | 1 | | 4 | 6 | | | | | | 5 | 8 |
| | | | | | | | | | 8 | | 5 | | | | | | | | | |
| | | | | | | | | 9 | | | | 2 | | | | | | | | |
| | | | | | | | | | 9 | | 6 | | | | | | | | | |
| 6 | 4 | | | | | | 9 | 8 | | 6 | | 5 | 4 | | | | | | 9 | 7 |
| 2 | | 8 | | | | 7 | | 5 | | | | 6 | | 3 | | | | 4 | | 1 |
| | 1 | 7 | | | | 4 | 3 | | | | | | 8 | 9 | | | | 3 | 2 | |
| | | | 3 | | 9 | | | | | | | | | | 1 | | 8 | | | |
| | | | | 6 | | | | | | | | | | | | 5 | | | | |
| | | | 5 | | 8 | | | | | | | | | | 6 | | 9 | | | |
| | 5 | 4 | | | | 1 | 6 | | | | | | 1 | 8 | | | | 9 | 7 | |
| 9 | | 1 | | | | 8 | | 7 | | | | 9 | | 7 | | | | 8 | | 2 |
| 7 | 3 | | | | | | 4 | 9 | | | | 2 | 3 | | | | | | 6 | 4 |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

$$= \frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}\overrightarrow{AB} + \frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}\overrightarrow{AC}$$

$$= \alpha\,\overrightarrow{AB} + \beta\,\overrightarrow{AC} \text{ (given)}$$

$$\therefore\ \alpha = \frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|} \text{ and } \beta = \frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$$

**5. (b) :** Let $x^3 = u$, then $3x^2dx = du$

Also, $\int f(x)dx = \psi(x)$. Now, $\int x^5 f(x^3)dx = \frac{1}{3}\int u\, f(u)du$

$$= \frac{1}{3}\left[u\int f(u)du - \int\left(\int f(u)du\right)du\right]$$

$$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$$

**6. (a) :** $16(\sin^5x + \cos^5x) - 11(\sin x + \cos x) = 0$

$\Rightarrow (\sin x + \cos x)\{16(\sin^4x - \sin^3x\cos x + \sin^2x\cos^2x - \sin x\cos^3x + \cos^4x) - 11\} = 0$

$\Rightarrow (\sin x + \cos x)\{16(1 - \sin^2x\cos^2x - \sin x\cos x) - 11\} = 0$

$\Rightarrow (\sin x + \cos x)(4\sin x\cos x - 1)(4\sin x\cos x + 5) = 0$

As $4\sin x\cos x + 5 \neq 0$, we have

$\sin x + \cos x = 0$ or $4\sin x\cos x - 1 = 0$

The required values are $\pi/12$, $5\pi/12$, $9\pi/12$, $13\pi/12$, $17\pi/12$, $21\pi/12$, – they are 6 solutions on $[0, 2\pi]$.

**7. (a) :** $\tan\theta/2 = \sqrt{\left(\frac{1-\cos\theta}{1+\cos\theta}\right)} = \sqrt{\frac{1-\left(\frac{a\cos\phi+b}{a+b\cos\phi}\right)}{1+\left(\frac{a\cos\phi+b}{a+b\cos\phi}\right)}}$

$$= \sqrt{\frac{(a-b)(1-\cos\phi)}{(a+b)(1+\cos\phi)}} = \sqrt{\frac{(a-b)}{(a+b)}}\tan(\phi/2)$$

**8. (a) :** $x^ky^kz^k\begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^2r^6 \end{vmatrix}$

$a^{3(k+1)}.\ r^{3(2k+1)}[r - 1](r^4 - 1) - (r^2 - 1)^2] \Rightarrow k = -1$

**9. (b) :** $5|x| - x^2 - 6 \geq 0 \Rightarrow x^2 - 5|x| + 6 \leq 0$

when $x < 0$, $x^2 + 5x + 6 \leq 0$, $-3 \leq x \leq -2$

when $x > 0$, $x^2 - 5x + 6 \leq 0$, $2 \leq x \leq 3$

$x = 0$ will not satisfy the condition.

Domain is $[-3, -2] \cup [2, 3]$.

**10. (a) :** Given: $f(x) = (-1)^{[x]}$.

First of all, we sketch the graph of $f(x)$ with the help of piecewise defined functions as follows:

$$f(x) = (-1)^{[x]} = \begin{cases} 1; & -2 \leq x < -1 \\ -1; & -1 \leq x < 0 \\ 1; & 0 \leq x < 1 \\ -1: & 1 \leq x < 2 \\ 1; & 2 \leq x < 3. \end{cases}$$

$\because$ The function $f(x)$ repeats its value after the least interval of 2.

Therefore, the function $f(x)$ is periodic with period 2.

**11. (8) :** Let $\alpha = \sin^{-1}\frac{\sqrt{5}}{3}$, then $\cos^{-1}\frac{\sqrt{5}}{3} = \frac{\pi}{2} - \alpha$

Hence the expression = $\sin\left\{2\left(\alpha - \frac{\pi}{2} + \alpha\right)\right\}$

$= \sin(4\alpha - \pi) = -\sin4\alpha = -4\sin\alpha\cos\alpha(1 - 2\sin^2\alpha)$

$$= (-4)\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)\left(\frac{-1}{9}\right) = \frac{8\sqrt{5}}{81}$$

**12. (8) :** $I = \int(x^9 + x^6 + x^3)(2x^6 + 3x^3 + 6)^{1/3}dx$

$$= \int x^2(x^6 + x^3 + 1)(2x^9 + 3x^6 + 6x^3)^{1/3}dx$$

Put $2x^9 + 3x^6 + 6x^3 = t \Rightarrow 18(x^8 + x^5 + x^2)dx = dt$

$\Rightarrow 18x^2(x^6 + x^3 + 1)dx = dt$

$\therefore\ I = \frac{1}{24}(2x^9 + 3x^6 + 6x^3)^{4/3} + K \quad \therefore\ \frac{AB}{4} = 8$

**13. (15) :** $np = 2$ ...(i)

$npq = 1$ ...(ii)

$\Rightarrow\ q = \frac{1}{2},\ p = \frac{1}{2},\ n = 4$ (from (i) and (ii))

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 1 - {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$$

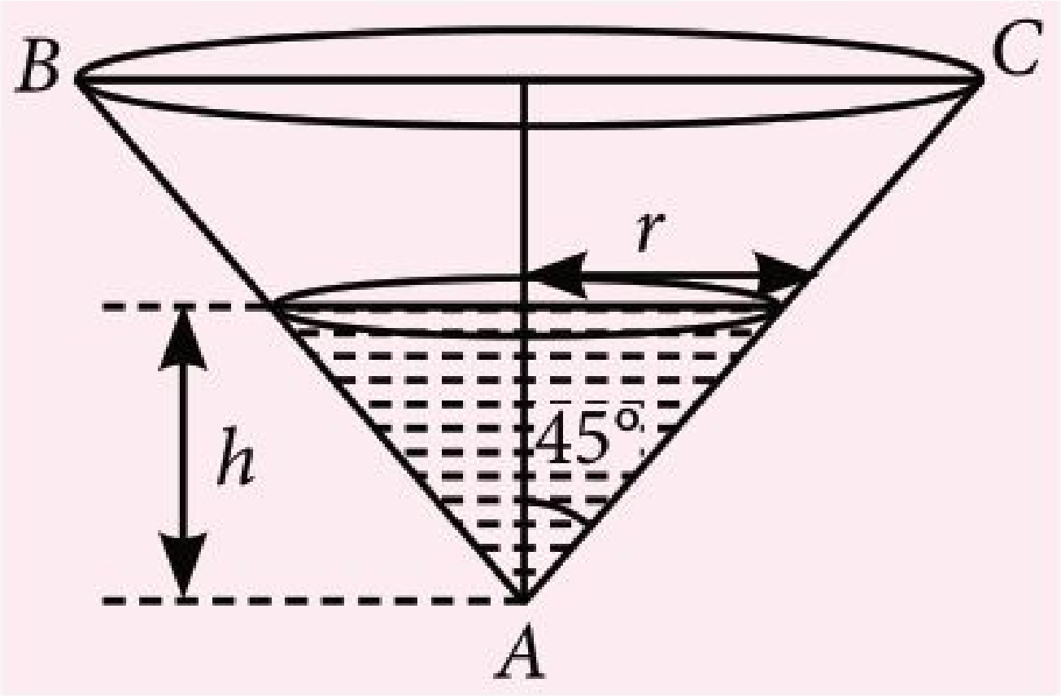
**14. (9) :** $\vec{r}\times\vec{b} = \vec{c}\times\vec{b} \Rightarrow (\vec{r} - \vec{c})\times\vec{b} = \vec{0}$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$ ...(i)

$\Rightarrow \vec{r}\cdot\vec{a} = \vec{a}\cdot\vec{c} + \lambda\vec{a}\cdot\vec{b} \Rightarrow \vec{r} = \vec{c} - \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\vec{b}$

$\Rightarrow \vec{r}\cdot\vec{b} = \vec{b}\cdot\vec{c} - \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\left(|\vec{b}|^2\right) = 1 - \frac{-4}{1}(2) = 9$

**15. (5) :** We have,



$\frac{dV}{dt} = 2$ or $\frac{d}{dt}\left(\frac{1}{3}\pi r^3\right) = 2$

$[\because r = h$ as $\theta = 45°]$

$\Rightarrow \pi r^2\frac{dr}{dt} = 2 \Rightarrow \frac{dr}{dt} = \frac{2}{\pi r^2}$ ...(i)

Now, perimeter = $2\pi r = p$ (say)

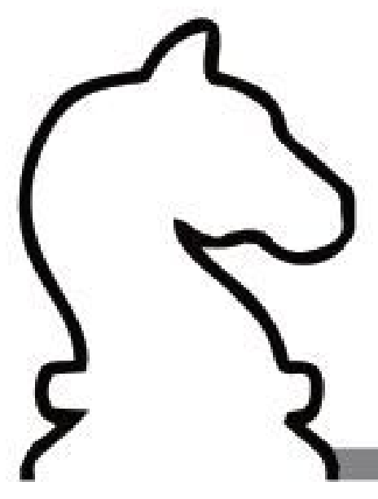
$\therefore\ \frac{d}{dt}(2\pi r) = 2\pi\frac{dr}{dt} = 2\pi\left(\frac{2}{\pi r^2}\right) = \frac{4}{r^2}$ ...(ii) [Using (i)]

When $h = 2$ m, $r = 2$ m.

Hence, $\frac{dp}{dt} = \frac{4}{4} = 1$ m/s.

◆ ◆

# Challenging PROBLEMS For JEE



ON

## Continuity & Differentiability

### SINGLE CORRECT ANSWER TYPE

**1.** Let $f(x)$ be a polynomial with real coefficients satisfies $f(x) = f'(x) \times f'''(x)$. If $f(x) = 0$ satisfies $x = 1, 2$ and 3 only, then the value of $f'(1) \times f'(2) \times f'(3)$ is

(a) positive (b) negative

(c) 0 (d) inadequate data

**2.** Which of the following functions are differentiable in (–1, 2) ?

(a) $\int\limits_{x}^{2x} (\log x)^2\, dx$ (b) $\int\limits_{x}^{2x} \frac{\sin x}{x}\, dx$

(c) $\int\limits_{0}^{x} \frac{1-t+t^2}{1+t+t^2}\, dt$ (d) None of these

**3.** If a function $f : [-2a, 2a] \to R$ is an odd function such that $f(2a - x) = f(x)$, $\forall\ x \in [a, 2a]$ and left hand derivative at $x = a$ is 0, then find left hand derivative at $x = -a$.

(a) 0 (b) –1 (c) 1/2 (d) $a$

**4.** If $f(x) = |x^2 + (k - 1)|\, x|-k|$ is non differentiable at five real points, then $k$ will lie in

(a) $(-\infty, 0)$ (b) $(0, \infty)$

(c) $(-\infty, 0) - \{-1\}$ (d) $(0, \infty) - \{1\}$

**5.** A function $f : R \to [1, \infty)$ satisfies the equation $f(xy) = f(x)\, f(y) - f(x) - f(y) + 2$. If $f$ is differentiable on $R - \{0\}$ and $f(2) = 5$, $f'(x) = \frac{f(x)-1}{x}\lambda$, then $\lambda =$

(a) $2f'(1)$ (b) $3f'(1)$

(c) $\frac{1}{2}f'(1)$ (d) $f'(1)$

**6.** Total number of points of non-differentiability of $f(x) = [3 + 4 \sin x]$ in $[\pi, 2\pi]$ where $[\cdot]$ denote the G.I.F, is

(a) 5 (b) 6 (c) 8 (d) 9

**7.** Let $f(x)$ be a differentiable function for all $x \in R$ and $f'(0) = 1$ then $g(x) = f(|x|) - \sqrt{\frac{1-\cos 2x}{2}}$

(a) is differentiable at $x = 0$ and its value is 1

(b) is differentiable at $x = 0$ and its value is 0

(c) is non differentiable at $x = 0$ as its graph has sharp turn at $x = 0$

(d) is non differentiable at $x = 0$ as its graph has vertical tangent at $x = 0$

**8.** If

$$f(x) = \begin{cases} \ln(1+|x|+(b-2)|x+1|) + a\tan^{-1} x + 2, & -2 < x < 0 \\ b, & x = 0 \\ \dfrac{(c+2)\sin^{-1}\sqrt{\{x\}} + e^{2x} - e^{\ln\left(4\left\{\frac{x}{2}\right\}\right)} - 1}{x^2}, & 0 < x < 2 \end{cases}$$

is derivable in (–2, 2) then find the value of $(9a^2 + b^2 + c^2)$. [$\{y\}$ denotes fractional part of $y$]

(a) 19 (b) 38 (c) 57 (d) 72

**9.** If $f(x) = \lim\limits_{m\to\infty} \frac{\log(e^m + x^m)}{m}$, $x \geq 0$ then

(a) $f(x)$ is discontinuous on $[0, \infty)$

(b) $f(x)$ is non-differentiable on $[0, \infty)$

(c) Area bounded by $f(x)$ between $x = 0$, $x = e$ and $x$-axis is 2 sq. units

(d) $f(x)$ is a periodic function in $[0, \infty)$

**10.** Let $f(x) = \begin{cases} x^p \sin\left(\frac{1}{x}\right) + x\left|x^3\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$

the set of values of $p$ for which $f''(x)$ is continuous at $x = 0$ is

(a) $[2, \infty)$ (b) $[-3, \infty)$

(c) $[5, \infty)$ (d) None of these

**By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105**

## MULTIPLE CORRECT ANSWER TYPE

**11.** If $f(x)=\begin{cases} x^2(\text{sgn}[x])+\{x\}, & 0\le x\le 2 \\ \sin x+|x-3|, & 2\le x<4 \end{cases}$, where $[\cdot]$ and $\{\cdot\}$ represents integer and fractional part functions, respectively, then

(a) $f(x)$ is differentiable at $x = 1$

(b) $f(x)$ is continuous but non-differentiable at $x = 1$

(c) $f(x)$ is non differentiable at $x = 2$

(d) $f(x)$ is discontinuous at $x = 2$

**12.** Let $f:\left[0,\frac{\pi}{2}\right]\to R$ be defined by $f(x)=\max\left\{\sin x,\cos x,\frac{3}{4}\right\}$ then which of the following is not true.

(a) $f(x)$ is continuous for all real $x \in [0, \pi/2]$

(b) $f(x)$ is not differentiable at $x=\frac{\pi}{4}$

(c) Number of points where $f(x)$ is not differentiable is 2

(d) Number of points where $f(x)$ is not continuous is 3

**13.** Let $f(x) = 6(x - 1)^{1/3} - 15(x - 1)^{2/3} + 12|x|$ on the interval $[-7, 2]$ then which of the following is/are correct?

(a) $f(x)$ is not differentiable at $x = 1$

(b) greatest value of $f(x)$ is 15

(c) least value of $f(x)$ is $-21$

(d) $f(x)$ has a local minimum at $x=\frac{9}{8}$

**14.** Given $f(x)=-\frac{x^3}{3}+x^2\sin 1.5a - x\sin a\cdot\sin 2a - 5\sin^{-1}(a^2 - 8a + 17)$, then

(a) $f'(x) = -x^2 + 2x\sin 6 - \sin 4\sin 8$

(b) $f'(\sin 8) > 0$

(c) $f'(x)$ is not defined at $x = \sin 8$

(d) $f'(\sin 8) < 0$

**15.** Let $f(x)=\int_0^x|\sin t|\,dt$, then

(a) $f$ is continuous everywhere

(b) $f'(x)$ is differentiable at $x = \pi$

(c) $f(\pi) = 2$

(d) $f(x) \ge 0$ for all real $x$

**16.** Let $f(x)=\begin{cases} x+2, & 0\le x<2 \\ 6-x, & x\ge 2 \end{cases}$,

$g(x)=\begin{cases} 1+\tan x, & 0\le x<\frac{\pi}{4} \\ 3-\cot x, & \frac{\pi}{4}\le x<\pi \end{cases}$

(a) $fog(x)$ is continuous in $[0, \pi)$

(b) $fog(x)$ is not continuous in $[0, \pi)$

(c) $fog(x)$ is differentiable in $[0, \pi)$

(d) $fog(x)$ is not differentiable in $[0, \pi)$

**17.** If $f(x)$ be such that $f(x) = \max\{|2 - x|, 2 - x^3\}$ then

(a) $f(x)$ is continuous $\forall\ x \in R$

(b) $f(x)$ is differentiable $\forall\ x \in R$

(c) $f(x)$ is non-differentiable at one point only

(d) $f(x)$ is non-differentiable at 4 point only

## COMPREHENSION TYPE

Let $f: R \to R$ be a differentiable function satisfying $f\left(\frac{x+y}{3}\right)=\frac{2+f(x)+f(y)}{3}$ for all real $x$ and $y$ and $f'(2) = 2$.

**18.** If $g(x) = |f(|x|) - 3|$ for all $x \in R$, then $g(x)$ has

(a) one non-differentiable point

(b) two non-differentiable points

(c) three non-differentiable points

(d) four non-differentiable points

**19.** If $x \in [-2, 3]$, then range of $f(x)$ is

(a) $[-2, 3]$ (b) $[-3, 4]$ (c) $[-2, 8]$ (d) $[-3, 10]$

**20.** The number of solutions of the equation $x^2 + (f\,|x|)^2 = 9$ is

(a) 0 (b) 2 (c) 3 (d) 5

## SOLUTIONS

**1.** **(c) :** $f(x) = f'(x) \times f'''(x)$ is satisfied by only the polynomial of degree 4.

Since $f(x) = 0$ satisfies $x = 1, 2, 3$ only. It is clear one of the root is twice repeated.

$\Rightarrow\ f'(1)\,f'(2)\,f'(3) = 0$

**2.** **(c) :** $f(x)=\int_x^{2x}(\log x)^2\,dx$

$\Rightarrow\ f'(x) = 2(\log 2x)^2 - (\log x)^2$

$\therefore\ f'(x)$ does not exist for all $x$ in $(-1, 2)$

Let $g(x)=\int_x^{2x}\frac{\sin x}{x}dx \ \Rightarrow\ g'(x)=2\left(\frac{\sin 2x}{2x}\right)-\frac{\sin x}{x}$

$\therefore$ $g'(x)$ does not exist at $x = 0$ and so it is not differentiable in $(-1,2)$.

Let $h(x)=\int_0^x \left(\frac{1-t+t^2}{1+t+t^2}\right)dt$

then, $h'(x)=\frac{1-x+x^2}{1+x+x^2}$, which is defined for all $x$ in $(-1, 2)$ as $1 + x + x^2 \neq 0$.

**3. (a) :** $f'(-a^-)=\underset{h\to 0}{\text{Lt}}\ \frac{f(-a)-f(-a-h)}{h}$

$$\underset{h\to 0}{\text{Lt}}\ \frac{-f(-a)+f(a+h)}{h}=\underset{h\to 0}{\text{Lt}}\ \frac{-f(a)+f(a-h)}{h}=0$$

**4. (c) :** $f(x) = |x^2 + (k - 1)|x| - k| = |(|x| - 1)(|x| + k)|$

Both the roots of $(x - 1)(x + k) = 0$ should be positive and distinct.

$\Rightarrow$ $k \in (-\infty, 0) - \{-1\}$

**5. (d) :** $f'(x)=\underset{x\to 0}{\text{Lt}}\ \frac{f(x+h)-f(x)}{h}$

$$=\underset{x\to 0}{\text{Lt}}\ \frac{f\left\{x\left(1+\frac{h}{x}\right)\right\}-f(x)}{h}\ (x\neq 0 \text{ given})$$

$$=\underset{x\to 0}{\text{Lt}}\ \frac{f\left(1+\frac{h}{x}\right)-2}{h/x}\cdot\frac{f(x)-1}{x}$$

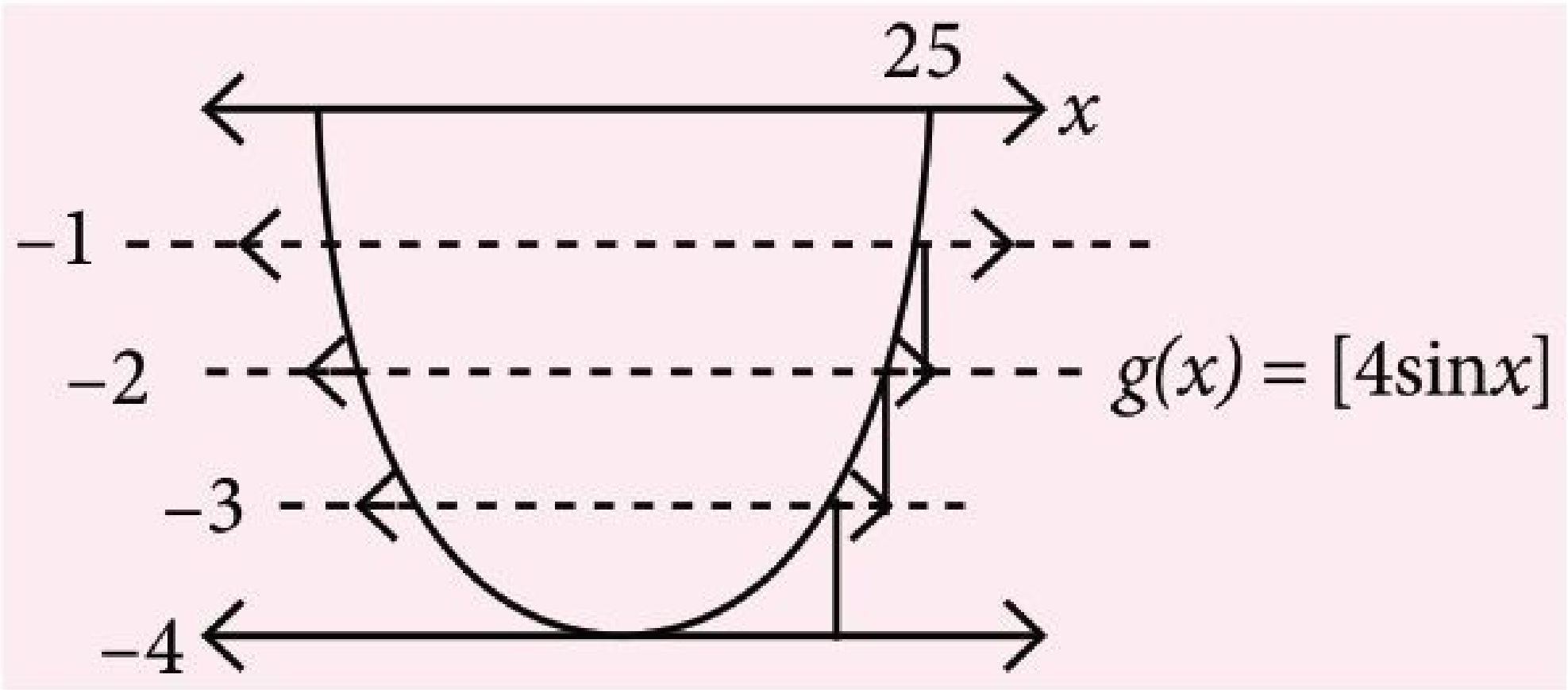
Putting $x = 1$, $y = 2$ in the given functional equation, $f(1) = 2$

$$\therefore\ f'(x)=\frac{f(x)-1}{x}\cdot\underset{h\to 0}{\text{Lt}}\ \frac{f\left(1+\frac{h}{x}\right)-f(1)}{h/x}=\frac{f(x)-1}{x}\cdot f'(1)$$

**6. (c) :** $f(x) = 3 + [4 \sin x]$

$f(x)$ is non-differentiable where $g(x) = [4 \sin x]$ is non differentiable.

In $[\pi, 2\pi]$, $g(x)$ is clearly non-differentiable at 8 points.



**7. (b) :** $g(x) = f(|x|) - |\sin x|$

$$g'(0^+)=\underset{h\to 0}{\text{Lt}}\ \frac{f(|h|)-|\sin h|-f(0)}{h}$$

$$=\underset{h\to 0}{\text{Lt}}\ \frac{f(h)-f(0)}{h}-\underset{h\to 0}{\text{Lt}}\ \frac{\sin h}{h}=1-1=0$$

$$g'(0^-)=\underset{h\to 0}{\text{Lt}}\ \frac{f(|-h|)-|\sin(-h)|-f(0)}{-h}$$

$$=\underset{h\to 0}{\text{Lt}}\ \frac{f(h)-f(0)}{-h}+\underset{h\to 0}{\text{Lt}}\ \frac{\sin h}{h}=-1+1=0$$

Thus $g(x)$ is differentiable at $x = 0$ and $g'(0) = 0$

**8. (c) :** Since, $f(x)$ is derivable in $(-2, 2)$.

Derivability of $f(x)$ at $x = -1$ is possible only when $b = 2$

$\{\because$ $|x + 1|$ is non-derivable at $x = -1.\}$

Similarly derivability of $f(x)$ at $x = 1$ is possible only when $c + 2 = 0 \Rightarrow c = -2$.

$$\text{Now, } f(x)=\begin{cases}\ln(1+|x|)+a\tan^{-1}x+2, & -2<x<0\\ 2, & x=0\\ \dfrac{e^{2x}-e^{\ln\left(4\left\{\frac{x}{2}\right\}\right)}-1}{x^2}, & 0<x<2\end{cases}$$

At $x = 0$,

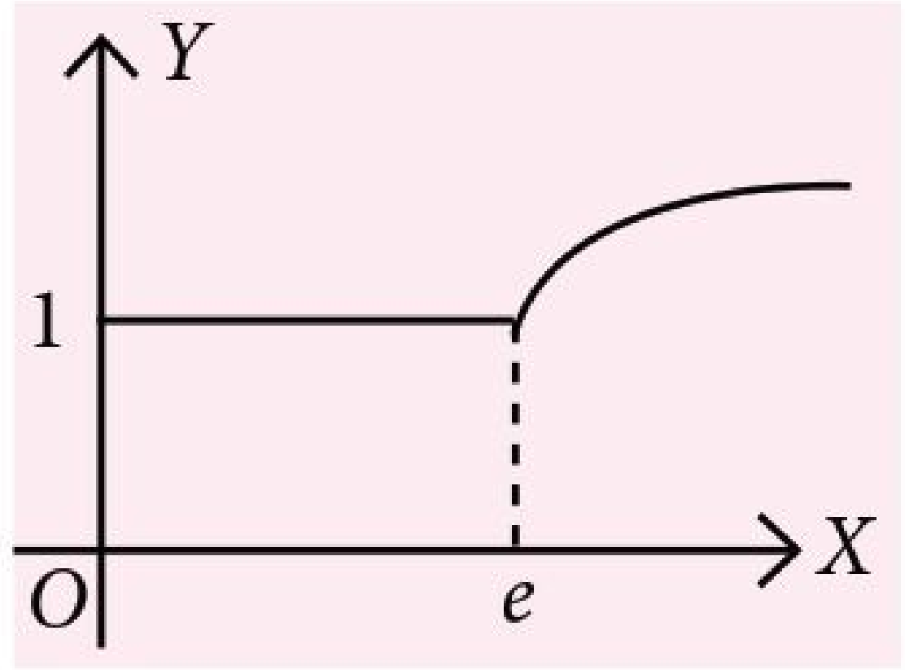
L.H.D. $= a - 1$ and R.H.D. $=\frac{4}{3}$

$$\therefore\ 9a^2+b^2+c^2=9\times\left(\frac{7}{3}\right)^2+(2)^2+(-2)^2=57$$

**9. (b) :** $f(x)=\lim\limits_{m\to\infty}\frac{\log(e^m+x^m)}{m}$

$$=\lim_{m\to\infty}\frac{m+\log\left(1+\left(\frac{x}{e}\right)^m\right)}{m}$$

$$=\begin{cases}1, \text{ if } 0\le x\le e\\ \log x, \text{ if } x>e\end{cases}$$



**10. (d) :** $f(x)=\begin{cases}x^p\sin\left(\frac{1}{x}\right)+x^4, & x>0\\ x^p\sin\left(\frac{1}{x}\right)-x^4, & x<0\\ 0, & x=0\end{cases}$

$$\Rightarrow\ f'(x)=\begin{cases}-x^{p-2}\cos\left(\frac{1}{x}\right)+px^{p-1}\sin\left(\frac{1}{x}\right)+4x^3, & x>0\\ -x^{p-2}\cos\left(\frac{1}{x}\right)+px^{p-1}\sin\left(\frac{1}{x}\right)-4x^3, & x<0\\ 0, & x=0\end{cases}$$

$$\therefore\ f''(x)=\begin{cases} -x^{p-4}\sin\left(\dfrac{1}{x}\right) & \\ -(p-2)x^{p-3}\cos\left(\dfrac{1}{x}\right) & \\ -px^{p-3}\cos\left(\dfrac{1}{x}\right) & x>0 \\ +p(p-1)x^{p-2}\sin\left(\dfrac{1}{x}\right)+12x^2, & \\ -x^{p-4}\sin\left(\dfrac{1}{x}\right) & \\ -(p-2)x^{p-3}\cos\left(\dfrac{1}{x}\right) & \\ -px^{p-3}\cos\left(\dfrac{1}{x}\right) & x<0 \\ +p(p-1)x^{p-2}\sin\left(\dfrac{1}{x}\right)-12x^2, & \\ 0, & x=0 \end{cases}$$

$\therefore$ sin ∞ and cos ∞ lies between –1 to 1.
For $p > 4$, R.H.L. = 0
L.H.L. = 0

$\therefore$ Value of function = 0

$\therefore$ For $p \in (4, \infty)$, $f''(x)$ is continuous.

**11. (b, c, d) :** For continuity at $x = 1$

$$\lim_{x\to1^+} f(x)=\lim_{x\to1^+} x^2\,\text{sgn}[x]+\{x\}=1+0=1$$

$$\lim_{x\to1^-} f(x)=\lim_{x\to1^-} x^2\,\text{sgn}[x]+\{x\}=0+1=1$$

Also $f(1) = 1$

Hence continuous at $x = 1$

$$f(x)=\begin{cases} x, & 0\le x<1 \\ x^2+x-1, & 1\le x<2 \end{cases}$$

$f(x)$ is not differentiable at $x = 1$.

$$\lim_{x\to2^-} f(x)=\lim_{x\to2^-} x^2\,\text{sgn}[x]+\{x\}=4\times1+1=5$$

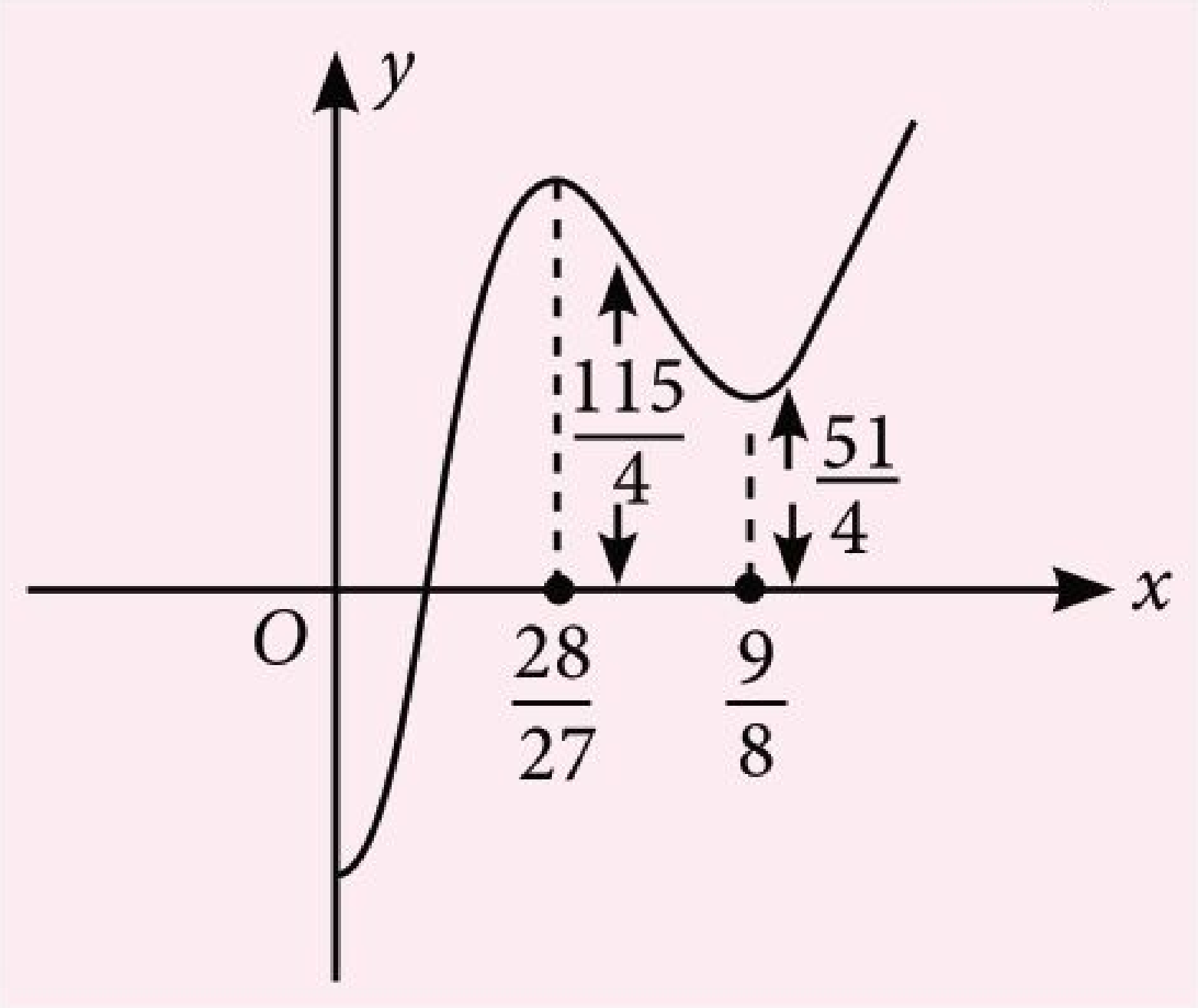
$$\lim_{x\to2^+} f(x)=\lim_{x\to2^+} \sin x+|x-3| = 1+\sin2$$

Hence $f(x)$ is discontinuous and hence non differentiable at $x = 2$.

**12. (b, d) :** $f(x)=\begin{cases} \cos x, & 0\le x\le \cos^{-1}(3/4) \\ 3/4, & \cos^{-1}(3/4)<x<\sin^{-1}(3/4) \\ \sin x, & \sin^{-1}(3/4)\le x\le \pi/2 \end{cases}$

$f(x)$ is continuous for all $x$ and is not differentiable at 2 points.

**13. (a,b,c,d) :** For $x > 0$, $f(x) = 6(x-1)^{1/3} - 15(x-1)^{2/3} + 12x$



$$f'(x)=\frac{2}{(x-1)^{2/3}}-\frac{10}{(x-1)^{1/3}}+12$$

$\therefore\ f'(x) = 0 \Rightarrow x=\dfrac{9}{8},\dfrac{28}{27}$

$x < 0 \Rightarrow f(x) = 6(x-1)^{1/3} - 15(x-1)^{2/3} - 12x$

$\Rightarrow\ f'(x)=\dfrac{2}{(x-1)^{2/3}}-\dfrac{10}{(x-1)^{1/3}}-12$

$f'(x)=0 \Rightarrow x=0, x=\dfrac{217}{216}$ (absurd)

**14. (a, d) :** $f(x)=-\dfrac{x^3}{3}+x^2\sin1.5a-x\sin a.\sin2a - 5\sin^{-1}(a^2-8a+17)$

$f(x)$ is defined, when $-1 \le a^2 - 8a + 17 \le 1$

$\Rightarrow\ -1 \le (a-4)^2 + 1 \le 1 \Rightarrow a = 4$

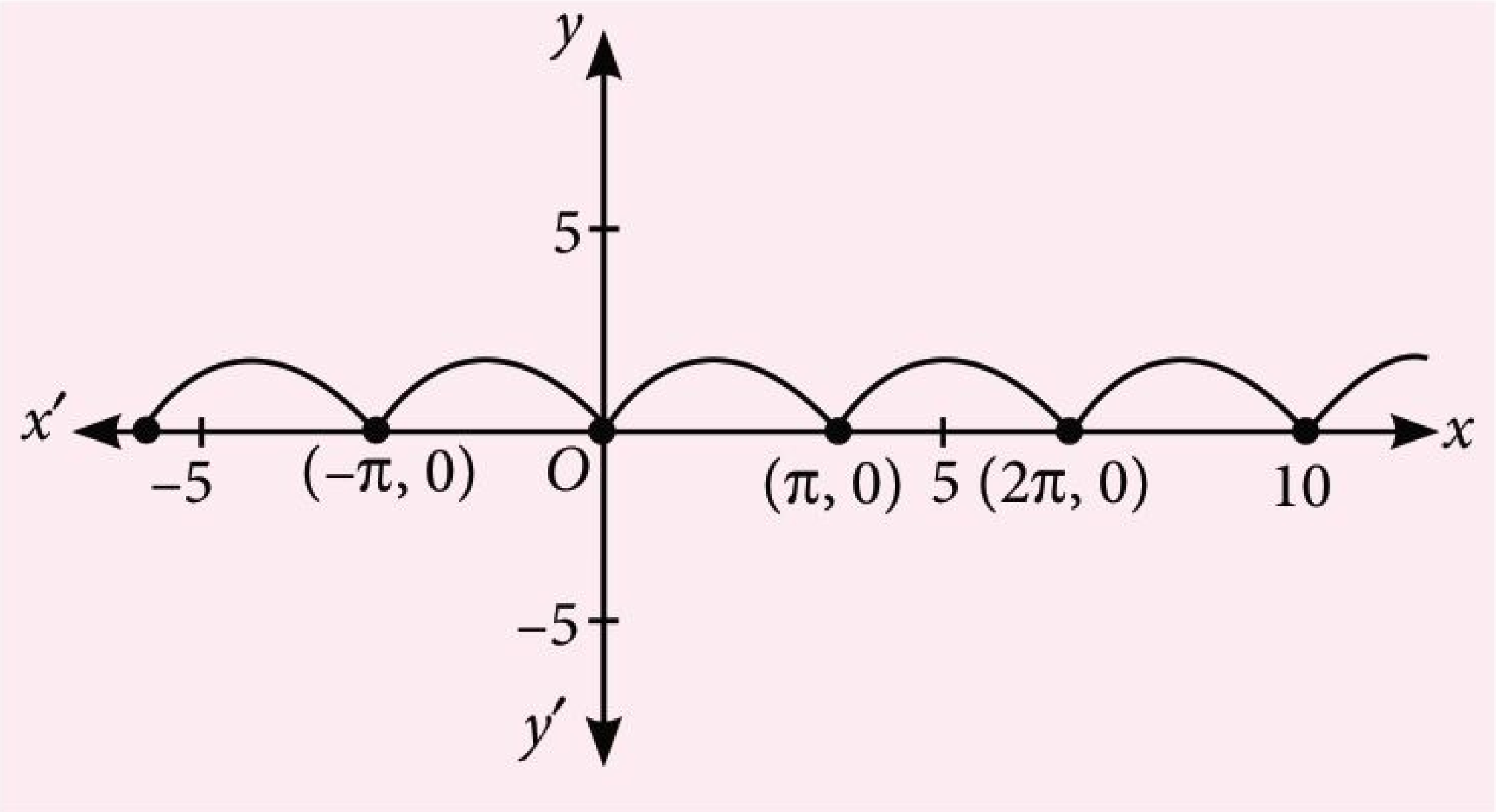
$\therefore\ f(x)=-\dfrac{x^3}{3}+x^2\sin6-x\sin4\sin8-\dfrac{5\pi}{2}$

$\therefore\ f'(x) = -x^2 + 2x\sin6 - \sin4\sin8$

$\Rightarrow\ f'(\sin8) = -\sin^2 8 + 2\sin8\sin6 - \sin4\sin8$
$= \sin8[2\sin6 - (\sin8 + \sin4)]$
$= \sin8\,[2\sin6 - 2\sin6\cos2]$
$= 2\sin6\sin8\,(1 - \cos2)$

$\Rightarrow\ \sin6 < 0, \sin8 > 0, 1 - \cos2 > 0 \quad \therefore f'(\sin8) < 0$

**15. (a, c, d) :** $f'(x) = |\sin x|$. Now draw the graphs.

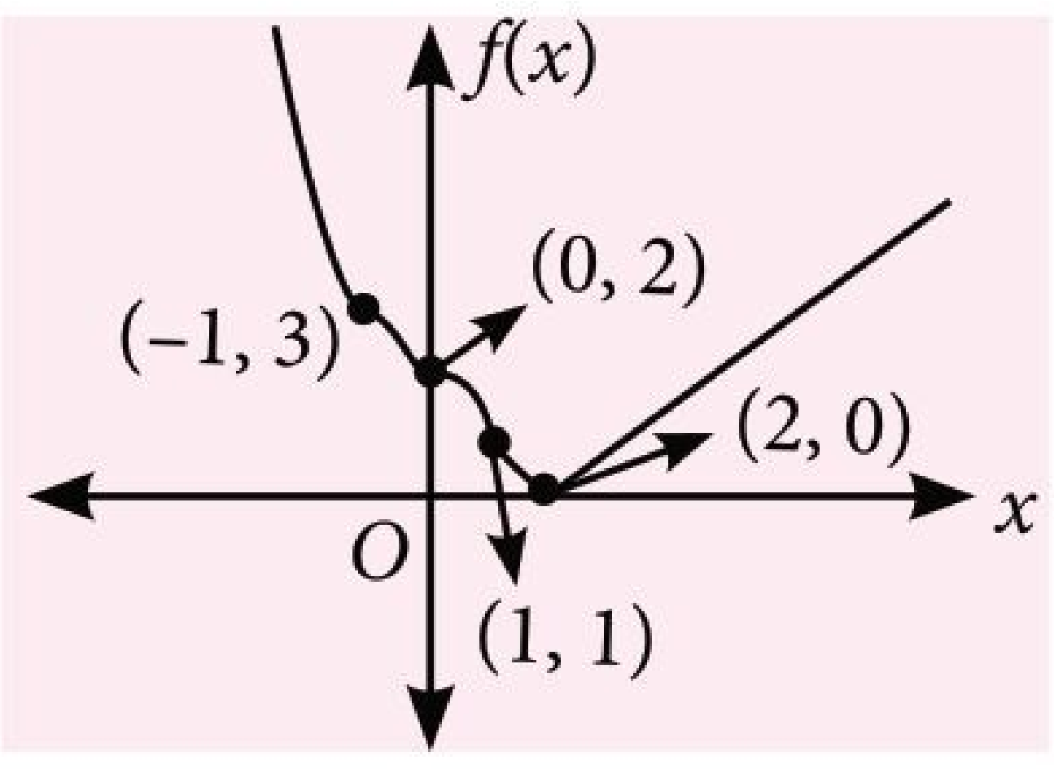


**16. (a, d) :** $fog(x)=\begin{cases} 3+\tan x, & 0\le x<\dfrac{\pi}{4} \\ 3+\cot x, & \dfrac{\pi}{4}\le x<\pi \end{cases}$

$fog(x)$ is continuous in $[0, \pi)$

but $fog(x)$ is not differentiable at $x = \dfrac{\pi}{4}$.

**17. (a, d) :** Clearly from the graph, $f(x)$ is continuous $\forall\, x \in R$ but not differentiable at $-1, 0, 1, 2$, (4 points).



**18. (c) :** $\because \quad f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{f\left(\dfrac{3x+3h}{3}\right) - f\left(\dfrac{3x+0}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{2+f(3x)+f(3h)}{3} - \dfrac{2+f(3x)+f(0)}{3}}{h}$$

$$= \lim_{h\to 0} \frac{f(3h)-f(0)}{3h} = f'(0)$$

$\Rightarrow \quad f'(2) = f'(0) = 2 \quad (\because f'(2) = 2)$

$\Rightarrow \quad f'(x) = 2 \Rightarrow f(x) = 2x + c$ ...(i)

Put $x = y = 0$ in $f\left(\dfrac{x+y}{3}\right) = \dfrac{2+f(x)+f(y)}{3}$

$\Rightarrow \quad f(0) = 2.$

Now, from (i), $f(0) = 0 + c = 2$

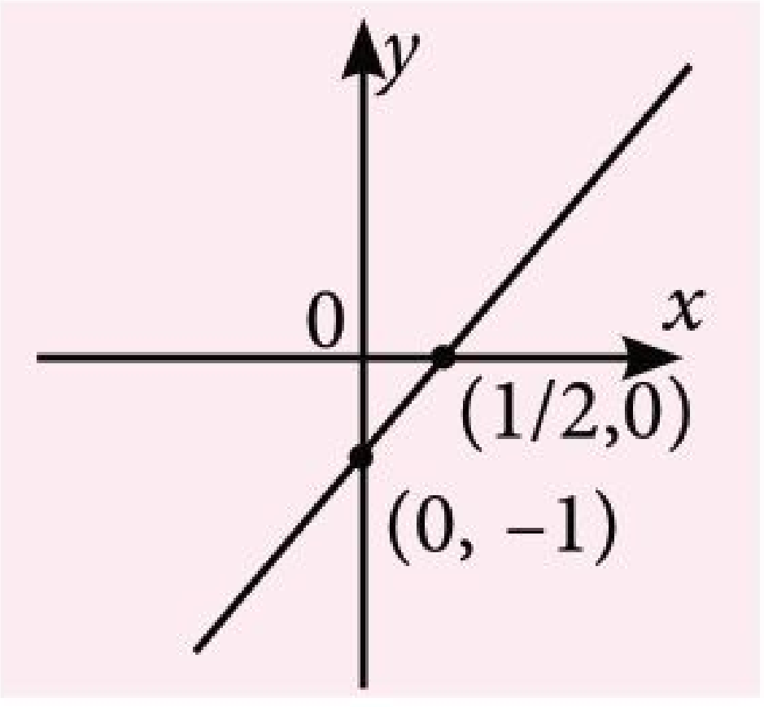
$\therefore \quad c = 2.$

From (i), $f(x) = 2x + 2.$

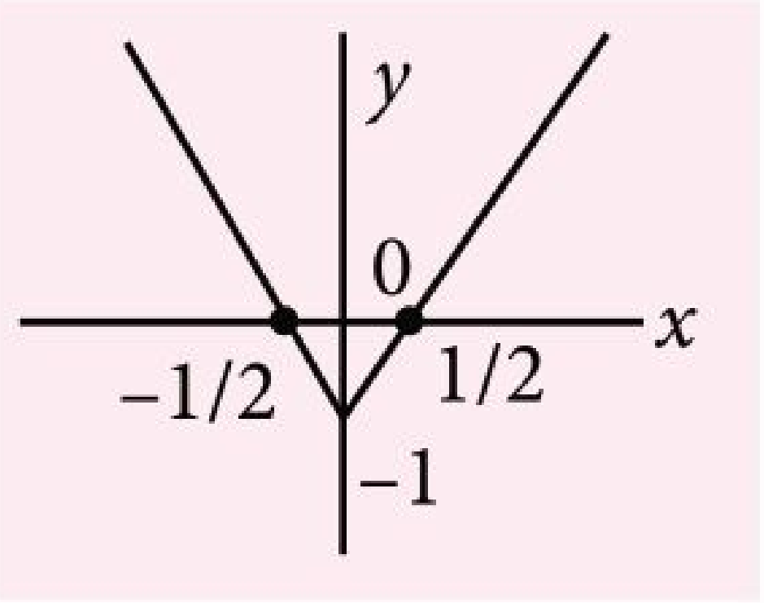
$\because \; f(|x|) = 2|x| + 2$

$\therefore \quad f(|x|) - 3 = 2|x| - 1$ and $g(x) = |f(|x|) - 3| = |2|x| - 1|$

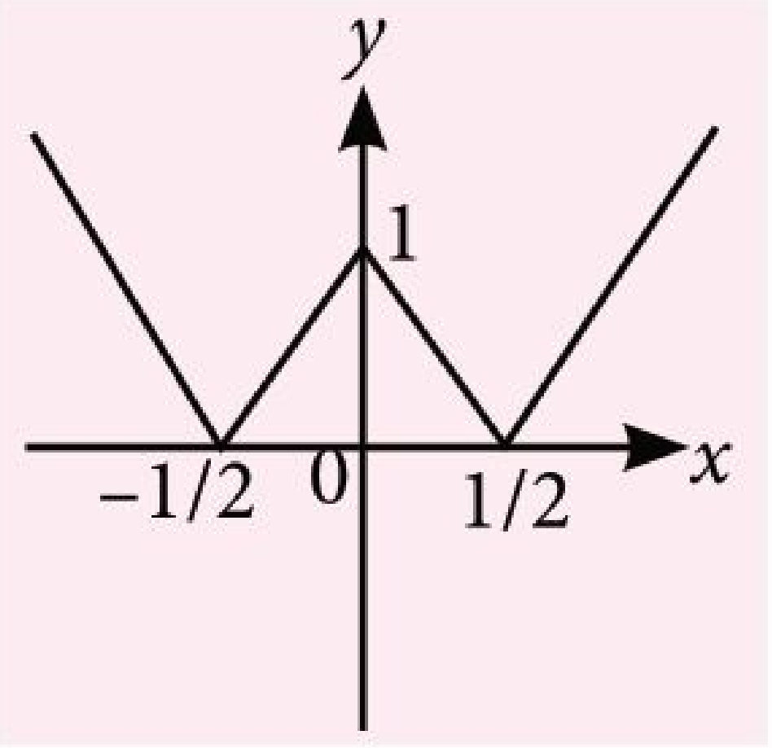
Now, graph of $y = 2x - 1$



Now, graph of $y = 2|x| - 1$



Graph of $g(x) = |2|x| - 1|$



Number of non-differentiable points = 3.

**19. (c) :** $\because \; f(x) = 2x + 2$

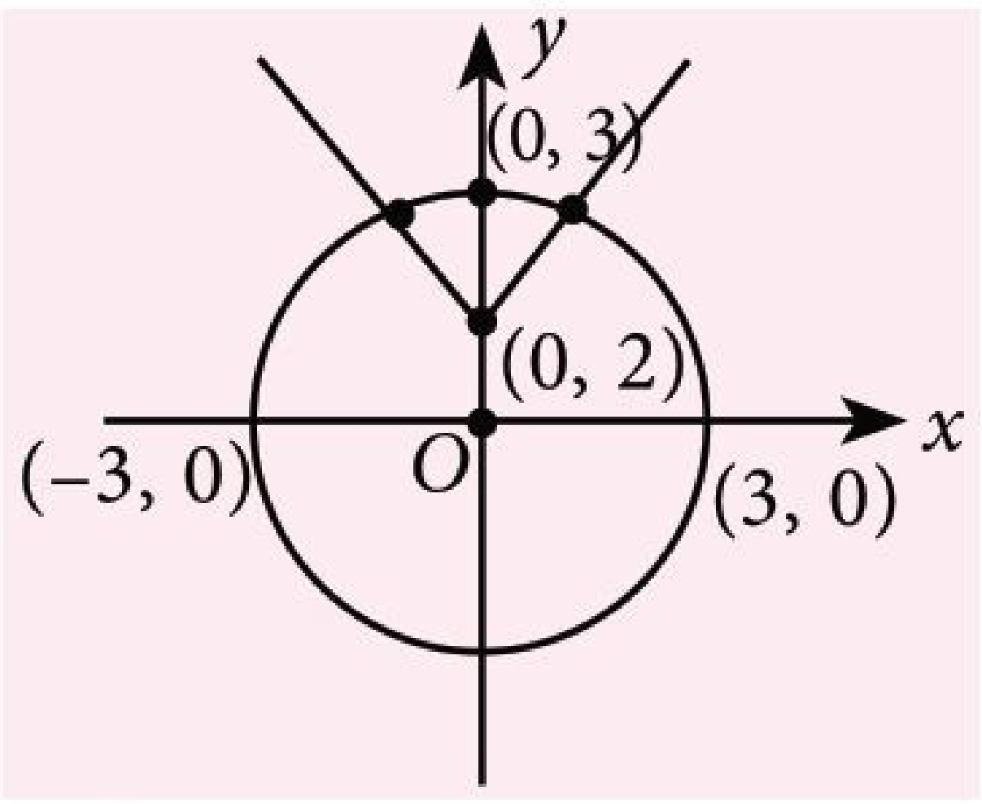
Here, given $x \in [-2, 3]$ or $-2 \le x \le 3$

$\Rightarrow \quad -4 \le 2x \le 6 \Rightarrow -4 + 2 \le 2x + 2 \le 6 + 2$

$\therefore \quad -2 \le f(x) \le 8$

Hence, range of $f(x)$ is $[-2, 8]$.

**20. (b) :** Let $y = f(|x|) = (2|x| + 2)$, then $x^2 + y^2 = 3^2$



$\therefore \quad$ Number of solutions = 2.

◆ ◆

# IIT Bombay most preferred for 93 of JEE's top 100

**COMPUTER SCIENCE MOST SOUGHT AFTER**

- IITs at **Delhi and Madras** were the **2nd & 3rd choices**
- **69 of top 100 will head to IIT-B**, up from last year's 62
- **28 to go to IIT-D,** 3 to IIT-M
- **Computer science was the most preferred** course at IIT Bombay this year.

The cream of the JEE toppers – the top 100 – has overwhelmingly picked IIT Bombay at the close of first-round admissions of 2022-23, making it the most sought-after tech college, with Delhi and Madras a distant second and third.

Ninety-three of them have opted for the Powai-based college as their first choice. All but one asked for computer science; 69 made the cut. Others had to make do with their second option: 28 of them will head to IIT Delhi and three of them to IIT Madras. Going by the final count, nine of the top 10 rankers will pursue B.Tech in computer science at IIT-B. A similar preference for the institute was shown by 48 of the top 50. The other two students opted for IIT Delhi and IIT Madras. Of the 48 who preferred IIT-B, 47 chose computer science and 1 chose engineering physics.

Last year, 62 of the top 100 joined IIT-B and 32 IIT Delhi. Several factors are responsible for this preference: from geography to gastronomy and placement records to what coaching classes tell students.

*Courtesy : The Times of India*

**Based on CBSE Circular released on 16th Sept. 2022**

# CBSE warm-up!

**CLASS-XII**

**Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.**

**Series-4**

## Application of Derivatives | Integrals | Application of Integrals

Time Allowed : 3 hours
Maximum Marks : 80

***General Instructions :***

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

### SECTION - A (MULTIPLE CHOICE QUESTIONS)

**Each question carries 1 mark.**

**1.** The side of a square increases at the rate of 1 cm per second. The rate at which the area of the square increases when the side is 15 cm is

(a) 27 sq. cm/sec
(b) 42 sq. cm/sec
(c) 35 sq. cm/sec
(d) 30 sq. cm/sec

**2.** Evaluate : $\int \frac{2x}{(2x+1)^2}dx$

(a) $\frac{1}{2}\log|2x+1| + \frac{1}{2(2x+1)} + c$
(b) $\frac{1}{2}\log|2x+1| - \frac{1}{2(2x+1)} + c$
(c) $2\log|2x+1| + \frac{1}{2(2x+1)} + c$
(d) $2\log|2x+1| - \frac{1}{2(2x+1)} + c$

**3.** For what values of $x$ is the rate of increase of $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of $x$ ?

(a) $-3, -\frac{1}{3}$ (b) $-3, \frac{1}{3}$
(c) $3, -\frac{1}{3}$ (d) $3, \frac{1}{3}$

**4.** The value of $\int \frac{1}{\cos 3x - \cos x}dx$ is

(a) $-\frac{1}{4}[\log|\sec x - \tan x| + \operatorname{cosec} x] + c$
(b) $-\frac{1}{4}[\log|\sec x + \tan x| - \operatorname{cosec} x] + c$
(c) $\frac{1}{4}\log[|\sec x - \tan x| - \operatorname{cosec} x] + c$
(d) $\frac{1}{4}[\log|\sec x + \tan x| + \operatorname{cosec} x] + c$

**5.** The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$, $x = 3$ is

(a) $\frac{7}{2}$ sq. units (b) $\frac{9}{2}$ sq. units
(c) $\frac{11}{2}$ sq. units (d) $\frac{13}{2}$ sq. units

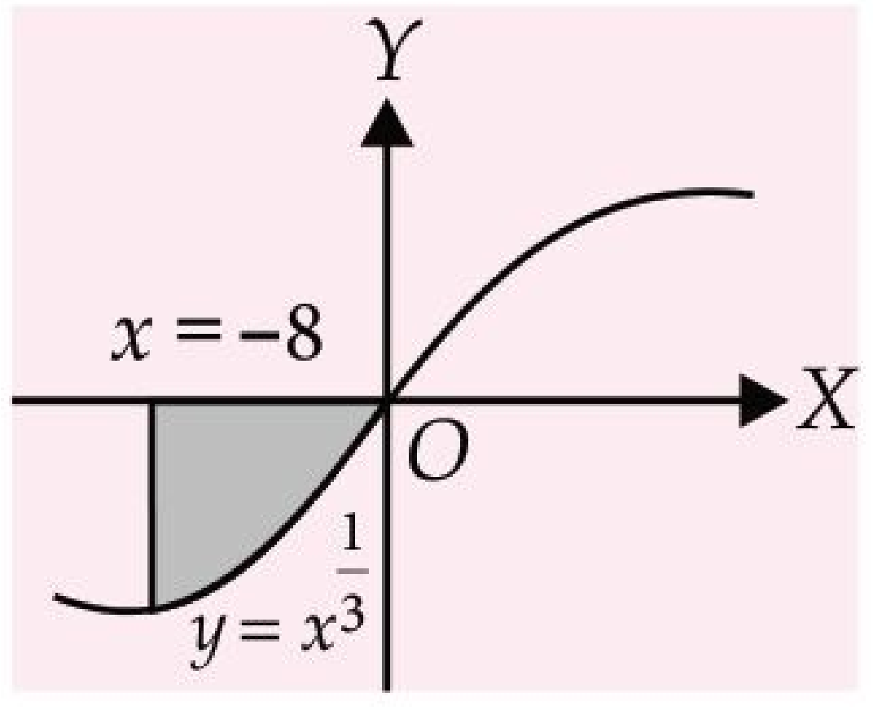
6. The maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is

(a) 0 (b) 12 (c) 16 (d) 32

7. Evaluate $\int_0^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x}$

(a) 1/2 (b) −2 (c) 2 (d) −1/2

8. Compute the shaded area shown in the given figure.



(a) 36 sq. units
(b) 12 sq. units
(c) 18 sq. units
(d) 15 sq. units

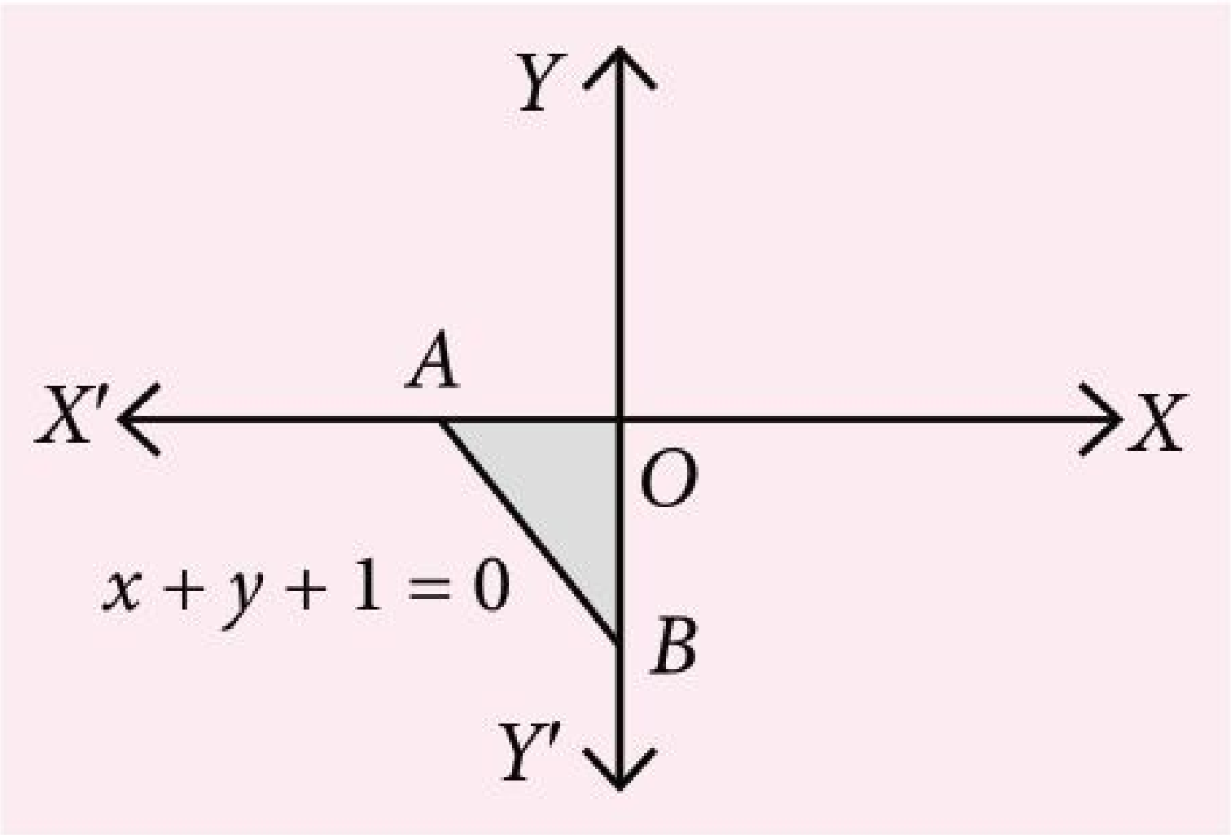
9. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

(a) $e^{-1/2}$ (b) $e^{1/e}$ (c) 1 (d) $e^2$

10. Evaluate $\int_{-2}^{2} \frac{x^3}{4-x^2} dx$ .

(a) $\frac{\pi}{2} - 1$ (b) 0
(c) $1 - \frac{\pi}{2}$ (d) $\log\left(\frac{2+e}{2-e}\right)$

11. Find the shaded area shown in the given figure.



(a) 1/4 sq. units
(b) 1/2 sq. units
(c) 3/2 sq. units
(d) 3 sq. units

12. Let the function $f : R \to R$ be defined by $f(x) = 2x + \cos x$, then $f$

(a) has a minimum at $x = \pi$
(b) has a maximum at $x = 0$
(c) is a decreasing function
(d) is an increasing function

13. Evaluate : $\int x \sin 2x \, dx$

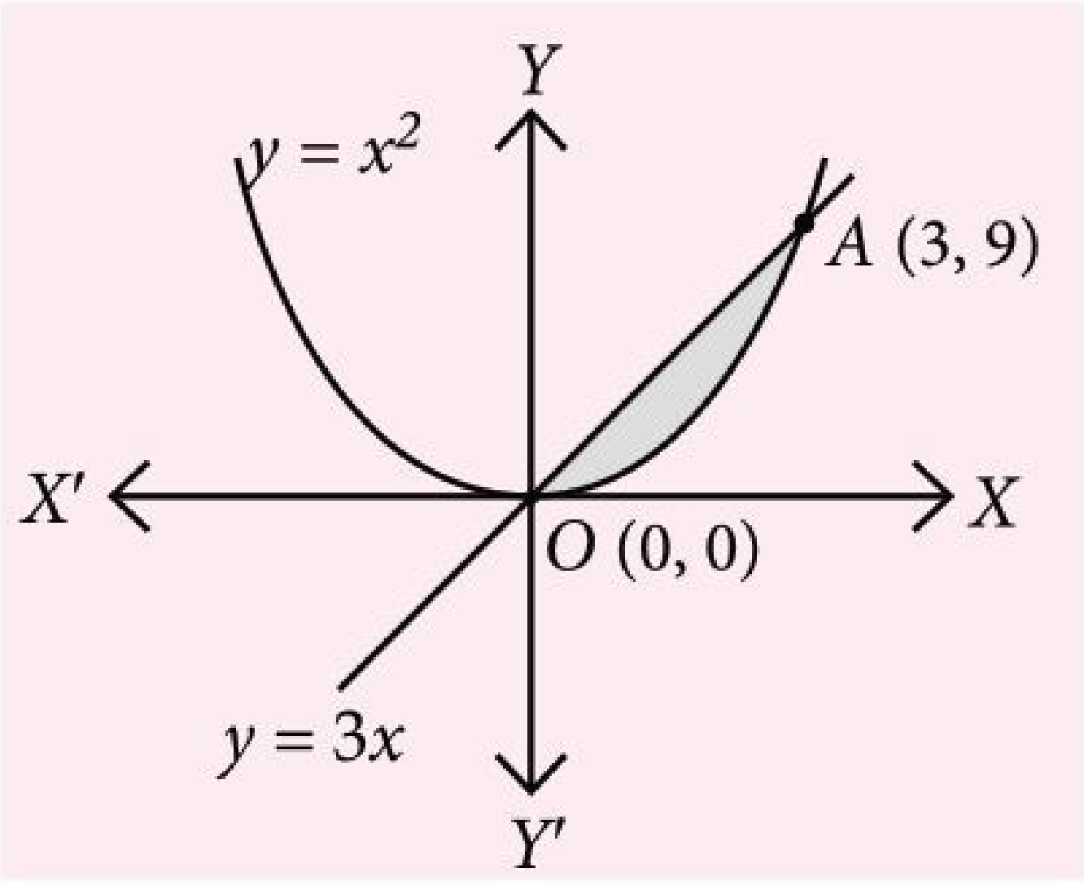
(a) $\frac{x \cos 2x}{2} + \frac{\sin 2x}{4} + C$

(b) $\frac{x \cos 2x}{2} - \frac{\sin 2x}{4} + C$

(c) $-\frac{x \cos 2x}{2} + \frac{\sin 2x}{4} + C$

(d) $\frac{x \cos 2x}{4} - \frac{\sin 2x}{2} + C$

14. Compute the shaded area shown in the given figure.



(a) 9 sq. units
(b) 4.5 sq. units
(c) 18 sq. units
(d) 7 sq. units

15. If $y = a \log x + bx^2 + x$ has its extremum at $x = -1$ and $x = 2$, then

(a) $a = -1/2, b = 2$ (b) $a = 2, b = 1/2$
(c) $a = 1/2, b = 1/2$ (d) $a = 2, b = -1/2$.

16. Find the area bounded by the curves $x = 1$, $x = 3$, $xy = 1$ and $X$-axis in sq. units.

(a) ln 3 (b) ln 2 (c) 0 (d) 3

17. The value of the integral $\int \frac{dx}{9x^2 - 8}$ is

(a) $\frac{1}{12\sqrt{2}} \log\left|\frac{3x-2\sqrt{2}}{3x+2\sqrt{2}}\right| + c$

(b) $\frac{1}{12\sqrt{2}} \log\left|\frac{3x+2\sqrt{2}}{3x-2\sqrt{2}}\right| + c$

(c) $\frac{1}{6\sqrt{2}} \log\left|\frac{3x-2\sqrt{2}}{3x+2\sqrt{2}}\right| + c$

(d) $\frac{1}{6\sqrt{2}} \log\left|\frac{3x+2\sqrt{2}}{3x-2\sqrt{2}}\right| + c$

18. The function $f(x) = \cot^{-1}x + x$ increases in the interval

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(0, \infty)$ (d) $(-\infty, \infty)$

## ASSERTION-REASON BASED QUESTIONS

**In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.**

**(a) Both A and R are true and R is the correct explanation of A.**
**(b) Both A and R are true but R is not the correct explanation of A.**
**(c) A is true but R is false.**
**(d) A is false but R is true.**

19. **Assertion (A) :** $\int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4033-x}} dx$ is equal to 1/2.

**Reason (R) :** $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$

20. **Assertion (A) :** $\int e^x(\tan x + \sec^2 x)dx = e^x \cdot \tan x + C$.

**Reason (R) :** $\int e^x\{f(x) + f'(x)\}dx = e^x f'(x) + C$

## SECTION - B

**This section comprises of very short answer type questions (VSA) of 2 marks each.**

**21.** Find the area of the curve $y = |\sin x|$ between 0 and $2\pi$.

**22.** Solve : $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$

**OR**

Find the value of $\int_{-1}^{1} \frac{|x|}{x} dx$.

**23.** Find : $\int \frac{x+1}{(x+2)(x+3)} dx$

**24.** Show that the function $x + 1/x$ is increasing for $x > 1$.

**OR**

Determine for which values of $x$, the function $y = x^4 - 4x^3/3$ is increasing and for which values, it is decreasing.

**25.** Find the area enclosed between the curve $y = \sqrt{x-1}$, the $X$-axis and the line $x = 5$.

## SECTION - C

**This section comprises of short answer type questions (SA) of 3 marks each.**

**26.** Show that for $a \geq 1$, $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing in $R$.

**27.** Find : $\int \sin^3 x \cos \frac{x}{2} dx$

**OR**

Evaluate the following : $\int \frac{3x - 1}{\sqrt{x^2 + 9}} dx$.

**28.** Evaluate : $\int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx$

**OR**

Find : $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$

**29.** Evaluate : $\int_0^{\infty} \frac{x^2}{(a^2 + x^2)^{5/2}} dx$

**30.** Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function on the set $R$. Find the relation between $a$ and $b$.

**31.** Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$.

**OR**

Find the area of the region bounded by $y = -1$, $y = 2$, $x = y^3$ and $x = 0$.

## SECTION - D

**This section comprises of long answer type questions (LA) of 5 marks each.**

**32.** Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

**33.** Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the ordinates $x = 0$ and $x = ae$, where $b^2 = a^2(1 - e^2)$ and $0 < e < 1$ (in square units).

**OR**

Draw the graph of $y = |x + 1|$ and using integration, find the area below $y = |x + 1|$, above $X$ - axis and between $x = -4$ to $x = 2$.

**34.** Find the local maximum and local minimum values of $f(x) = \sin 2x - x$, $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$.

**OR**

Let $AP$ and $BQ$ be two vertical poles at points $A$ and $B$ respectively. If $AP = 16$ m, $BQ = 22$ m and $AB = 20$ m, then find the distance of a point $R$ on $AB$ from the point $A$ such that $RP^2 + RQ^2$ is minimum.

**35.** Evaluate : $\int e^{2x} \sin(3x+1) dx$

## SECTION - E

**This section comprises of 3 case-study/passage-based question of 4 marks each with two sub-parts. First two case study question have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.**

**36.** **Case Study 1:** Integration is the process of finding the anti-derivative of a function. In this process, we are provided with the derivative of a function and asked to find out the function (*i.e.*, Primitive) Integration is the inverse process of differentiation. Let $f(x)$ be a function of $x$. If there is a function $g(x)$, such that $\frac{d}{dx}(g(x)) = f(x)$, then $g(x)$ is called an integral of $f(x)$ w.r.t. $x$ and is denoted by $\int f(x)dx = g(x) + c$, where $c$ is constant of integration. Based on the above information, answer the following questions.

**(i)** Evaluate : $\int \frac{(x+1)^2}{x(x^2+1)} dx$

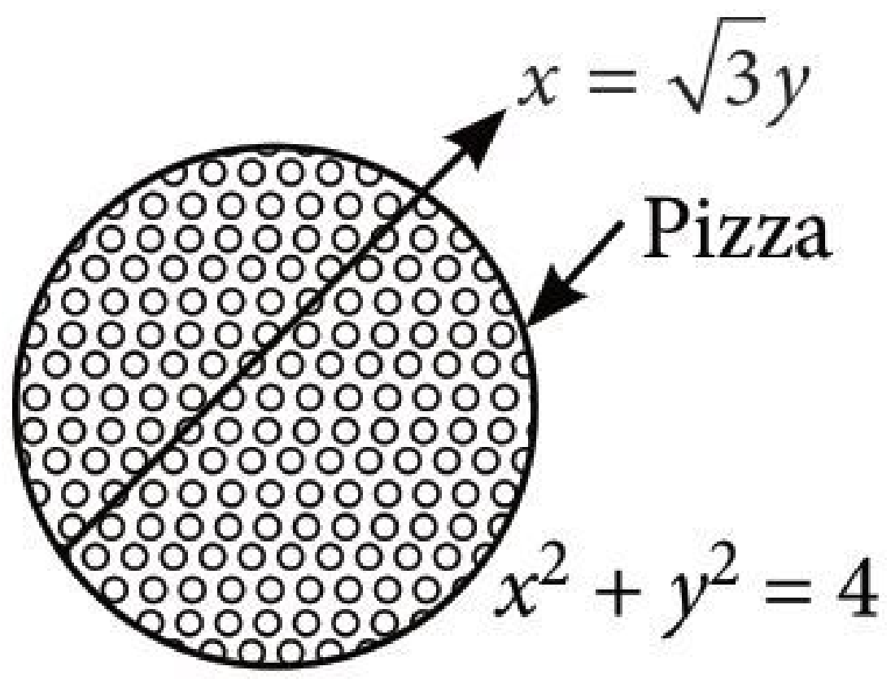
**(ii)** Evaluate : $\int \sin^2 x \, dx$

**(iii)** Evaluate : $\int \tan^2 x \, dx + \int \frac{dx}{\sin^2 x \cos^2 x}$

**OR**

Evaluate : $\int e^x \left(\frac{x+2}{x+4}\right)^2 dx$

**37.** **Case Study 2 :** A child cut a pizza with a knife. Pizza is circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of knife represents a straight line given by $x = \sqrt{3}y$.



Based on the above information, answer the following questions.

**(i)** Find the point(s) of intersection of the edge of knife (line) and pizza shown in the figure.

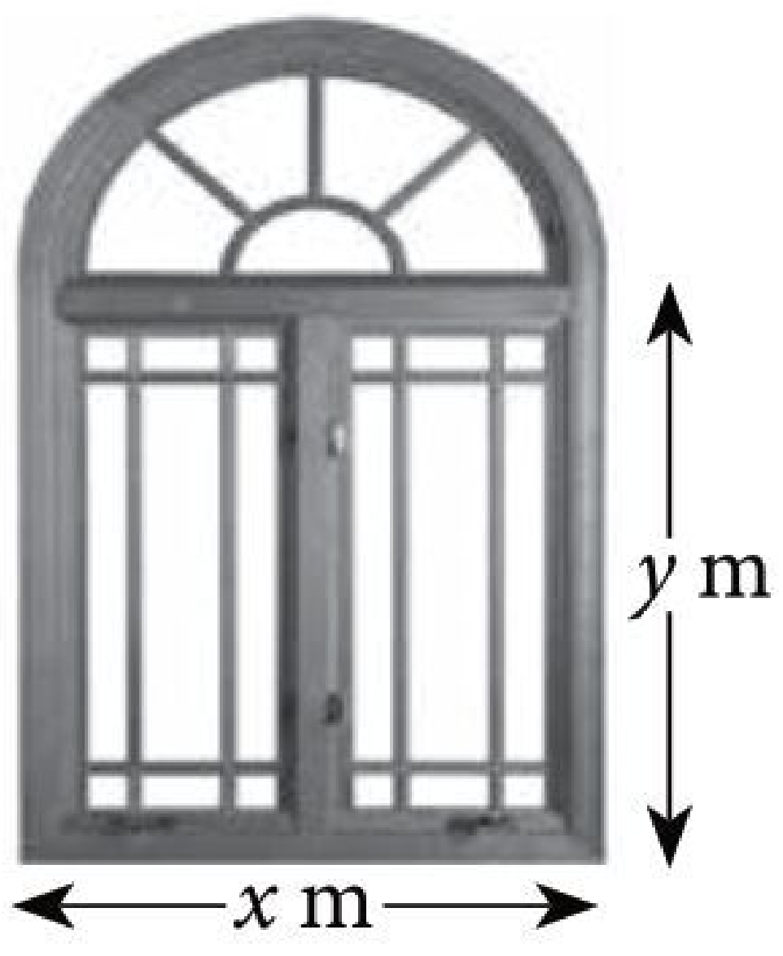
**(ii)** Represent the smaller area bounded by pizza and edge of knife in first quadrant.

**(iii)** Find the value of area of the region bounded by circular pizza and edge of knife in first quadrant.

**OR**

Find the area of each slice of pizza when child cut the pizza into 4 equal pieces. Find the slope of circle at point $(\sqrt{3}, 1)$.

**38.** **Case Study 3 :** Rohan, a student of class XII, visited his uncle's flat with his father. He observe that the window of the house is in the form of a rectangle surmounted by a semicircular opening having perimeter 10 m as shown in the figure.



Based on the above information, answer the following questions.

**(i)** Rohan is interested in maximizing the area of the whole window, for this to happen find the value of $x$.

**(ii)** Find the maximum area of the window. For maximum value of area, find the breadth of rectangular part of the window?

## SOLUTIONS

**1.** **(d) :** Area of square, $A = s^2$ (where $s$ is side of square)

$$\Rightarrow \frac{dA}{dt} = 2s\frac{ds}{dt} = 2\times 15\times 1 \text{ sq. cm/sec.} \left(\because \frac{ds}{dt} = 1 \text{ cm/sec}\right)$$

$$\Rightarrow \frac{dA}{dt} = 30 \text{ sq. cm/ sec}$$

**2.** **(a)**

**3.** **(d) :** Let $y = x^3 - 5x^2 + 5x + 8$

Given, $\frac{dy}{dt} = 2\frac{dx}{dt}$

$$\Rightarrow (3x^2 - 10x + 5)\frac{dx}{dt} = 2\frac{dx}{dt} \Rightarrow 3x^2 - 10x + 3 = 0$$

$$\Rightarrow 3x^2 - 9x - x + 3 = 0 \Rightarrow 3x(x-3) - 1(x-3) = 0$$

$$\Rightarrow (x-3)(3x-1) = 0 \Rightarrow x = 3 \text{ or } 1/3$$

**4.** **(b) :** Let $I = \int \frac{dx}{\cos 3x - \cos x} = \int \frac{\sin^2 x + \cos^2 x}{-4\sin^2 x \cos x} dx$

$$= -\frac{1}{4}\int (\sec x + \operatorname{cosec} x \cot x)\, dx$$

$$= -\frac{1}{4}[\log|\sec x + \tan x| - \operatorname{cosec} x] + c$$

**5.** **(a)**

**6.** **(b) :** We have, $y = -x^3 + 3x^2 + 9x - 27$

Let $s = \frac{dy}{dx} = -3x^2 + 6x + 9$. Now, $\frac{ds}{dx} = -6x + 6$

For critical points, $\frac{ds}{dx} = 0$

$$\Rightarrow -6x + 6 = 0 \Rightarrow x = 1 \;;\; \frac{d^2s}{dx^2} = -6 < 0$$

$\therefore$ Maximum value of $s$ at $x = 1$ is $(-3 + 6 + 9) = 12$

**7.** **(a)**

**8.** **(b) :** Required area

$$= \left|\int_{-8}^{0} x^{1/3}\, dx\right| = \left|\left[\frac{x^{4/3}}{4/3}\right]_{-8}^{0}\right| = \left|\frac{3}{4}[0 - (-8)^{4/3}]\right|$$

$$= \left|\frac{3}{4}[-(-2)^4]\right| = \frac{3}{4}\times 16 = 12 \text{ sq. units}$$

**9.** **(b) :** Let $y = \left(\frac{1}{x}\right)^{2x^2} \Rightarrow \log y = 2x^2 \log\frac{1}{x}$

On differentiating w.r.t. $x$, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = 4x\log\frac{1}{x} + 2x^2\left(-\frac{1}{x}\right)$$

*Recipe for Success*

SUCCESS

Is

the SUM of

SMALL

– efforts, –

Repeated

DAY IN AND DAY OUT

For maxima or minima

$\frac{dy}{dx}=0 \Rightarrow -4x\log x-2x=0\Rightarrow 2\log x+1=0\Rightarrow x=e^{-1/2}$

Now, $\left[\frac{d^2y}{dx^2}\right]_{x=e^{-1/2}}<0 \quad \therefore \; y_{\max}=e^{1/e}$

**10. (b)** **11. (b)**

**12. (d) :** We have, $f(x) = 2x + \cos x$

$\therefore \; f'(x) = 2 + (-\sin x) = 2 - \sin x$

$\therefore \; f'(x) > 0, \forall x \quad (\because -1 \le \sin x \le 1 \; \forall x \in R)$

**13. (c) :** Let $I=\int \underset{\text{I}}{x}\cdot\underset{\text{II}}{\sin 2x}\,dx$

$=x\int\sin 2x\,dx-\int\left[1\cdot\int\sin 2x\,dx\right]dx$

$=\frac{-1}{2}x\cos 2x+\frac{1}{2}\,\frac{\sin 2x}{2}+C=-\frac{x\cos 2x}{2}+\frac{\sin 2x}{4}+C$

**14. (b)**

**15. (d) :** We have, $y = a\log x + bx^2 + x$

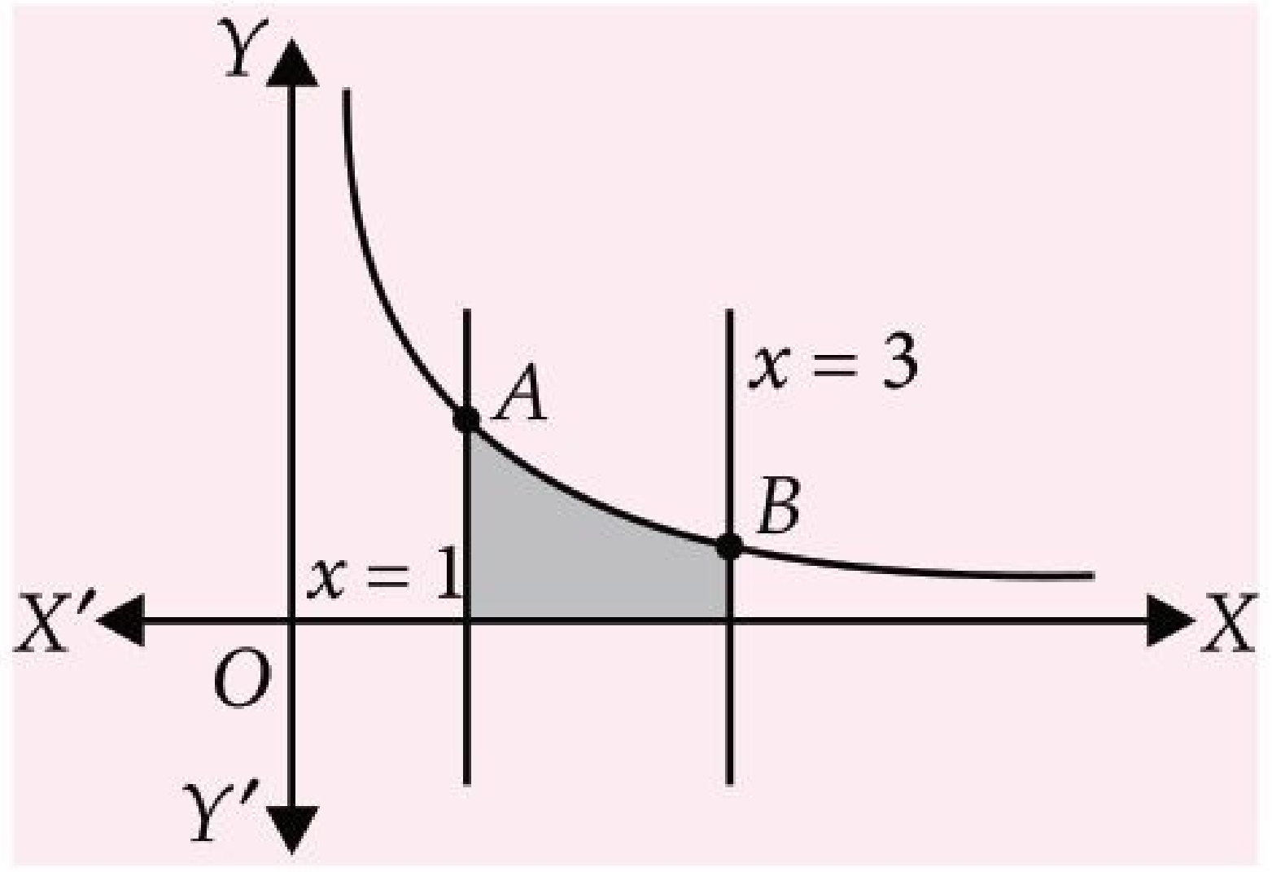
$\therefore \; \frac{dy}{dx}=\frac{a}{x}+2bx+1$

$\left(\frac{dy}{dx}\right)_{x=-1}=0 \Rightarrow a + 2b = 1$ ...(i)

and $\left(\frac{dy}{dx}\right)_{x=2}=0 \Rightarrow a + 8b = -2$ ...(ii)

Solving (i) & (ii), we get $a = 2$, $b = -1/2$

**16. (a) :** Given curves are $xy = 1$, $x = 1$, $x = 3$ and $X$-axis



$\therefore$ Required area = Area of shaded region

$=\int_1^3\left(\frac{1}{x}\right)dx=[\ln x]_1^3$ = (ln3 − ln1) = ln3 sq. units

**17. (a) :** Let $I=\int\frac{dx}{9x^2-8}=\frac{1}{9}\int\frac{dx}{x^2-\left(\frac{2\sqrt{2}}{3}\right)^2}$

$=\frac{1}{9}\times\frac{3}{2\left(2\sqrt{2}\right)}\log\left|\frac{3x-2\sqrt{2}}{3x+2\sqrt{2}}\right|+c=\frac{1}{12\sqrt{2}}\log\left|\frac{3x-2\sqrt{2}}{3x+2\sqrt{2}}\right|+c$

**18. (d) :** $f'(x)=1-\frac{1}{1-x^2}\ge 0$

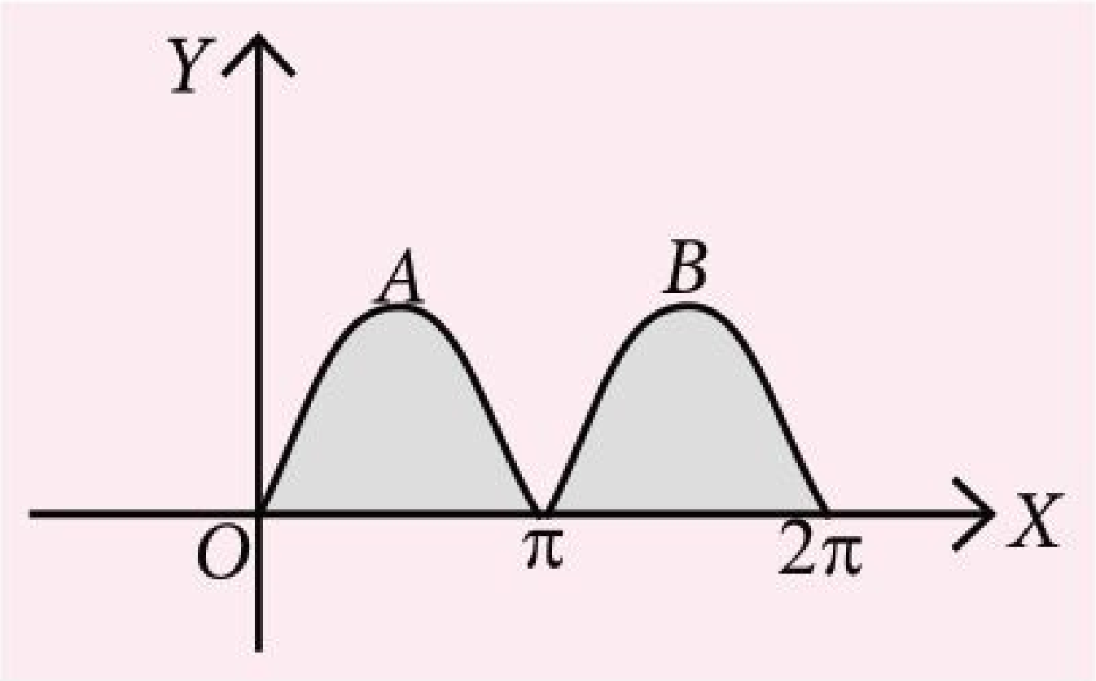
$\forall x\in R$

Here, $f(x)$ is increasing function $\forall x\in R$

**19. (a)** **20. (c)**

**21.** We have $y = |\sin x|$

$\therefore$ Required area = Area of shaded region



$=\int_0^{2\pi}\sin x\,dx=2\int_0^{\pi}\sin x\,dx$

$=2[-\cos x]_0^{\pi}$

$=-2[\cos\pi-\cos 0]$

$= -2[-1 -1] = 4$ sq. units

**22.** Let $I=\int_2^4\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$

$I=\int_2^4\frac{\log x^2}{\log x^2+\log(6-x)^2}dx$ ...(i)

By using $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, we get

$I=\int_2^4\frac{\log(6-x)^2}{\log(6-x)^2+\log x^2}dx$ ...(ii)

Adding (i) and (ii), we get $2I = \int_2^4 dx \Rightarrow I=1$

**OR**

First note that $\frac{|x|}{x}$ is discontinuous at $x = 0$.

$\therefore \int_{-1}^{1}\frac{|x|}{x}dx=\int_{-1}^{0}\frac{|x|}{x}dx+\int_{0}^{1}\frac{|x|}{x}dx=\int_{-1}^{0}\frac{-x}{x}dx+\int_{0}^{1}\frac{x}{x}dx$

$(\because -1\le x\le 0\Rightarrow |x| = -x$ and $0\le x\le 1\Rightarrow |x| = x)$

$=\int_{-1}^{0}-1dx+\int_{0}^{1}1dx=[-x]_{-1}^{0}+[x]_0^1$

$= -(0-(-1)) + (1-0) = -1+1 = 0.$

**23.** Let $I=\int\frac{x+1}{(x+2)(x+3)}dx=\int\frac{-1}{(x+2)}dx+2\int\frac{1}{(x+3)}dx$

$= -\log|x + 2| + 2\log|x + 3| + C$

**24.** Let $f(x) = x+\frac{1}{x}$. Then, $f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$

Now, $x > 1 \Rightarrow x^2 > 1 \Rightarrow \frac{x^2-1}{x^2}>0 \Rightarrow f'(x) > 0.$

Hence, $f(x)$ is increasing for $x > 1$.

**OR**

Here $f(x)=x^4-\frac{4x^3}{3}$

$\therefore \; f'(x) = 4x^3 - 4\times\frac{3x^2}{3} = 4x^3 - 4x^2 = 4x^2(x - 1)$

For $f(x)$ to be increasing, we must have $f'(x) > 0$

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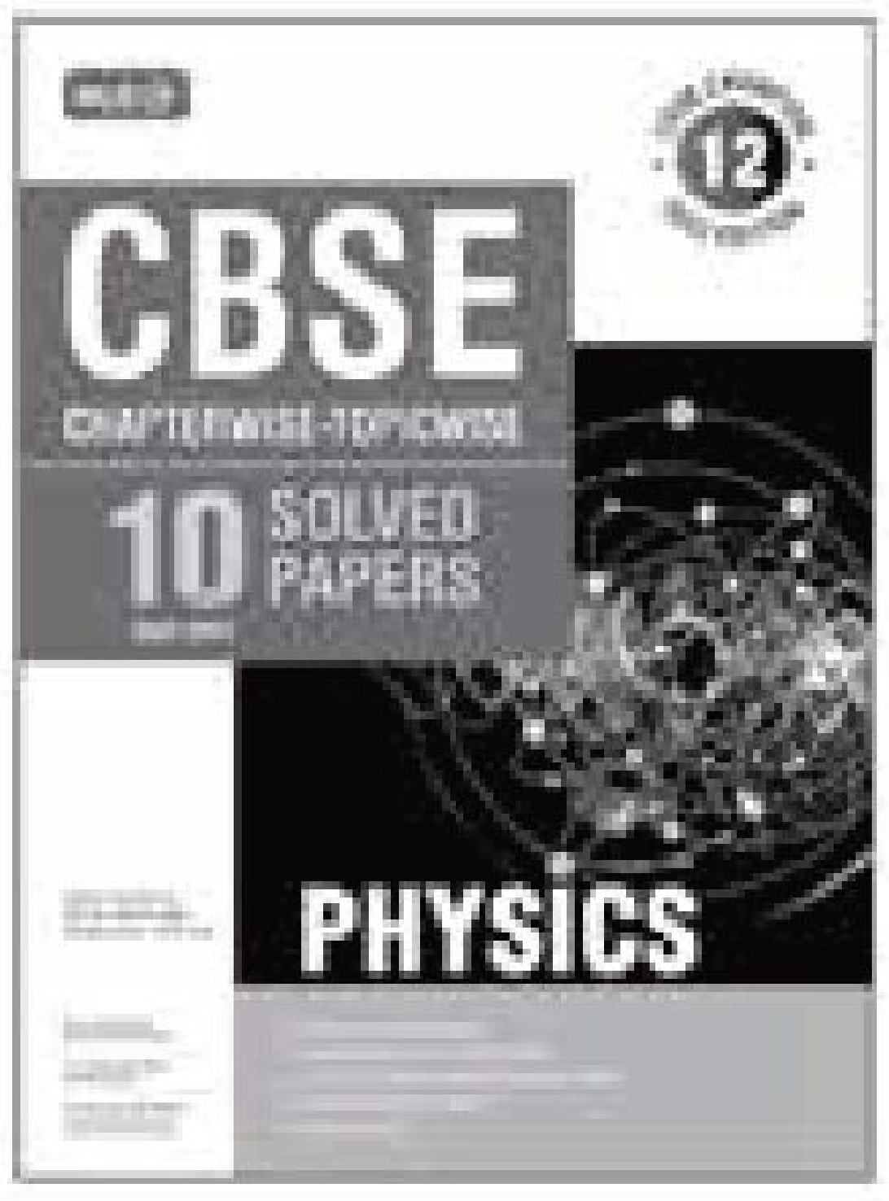
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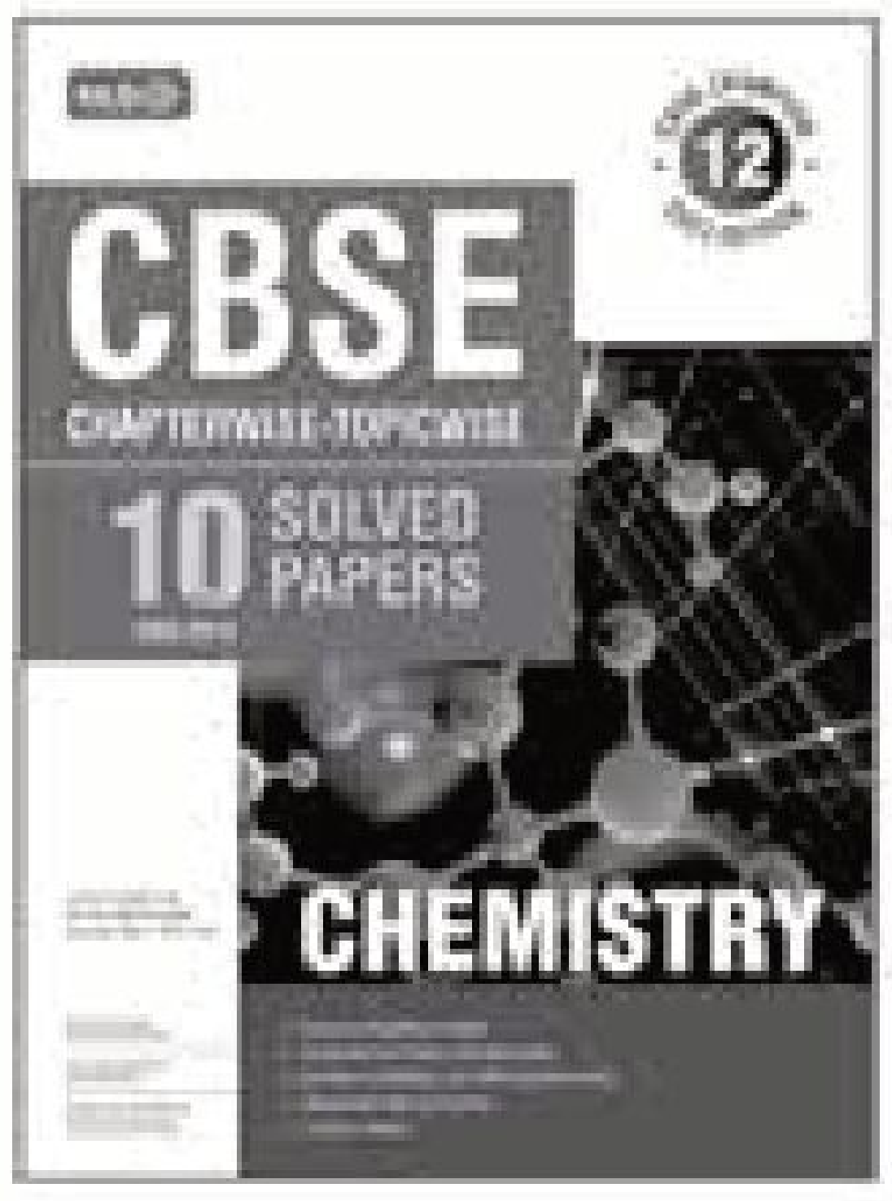
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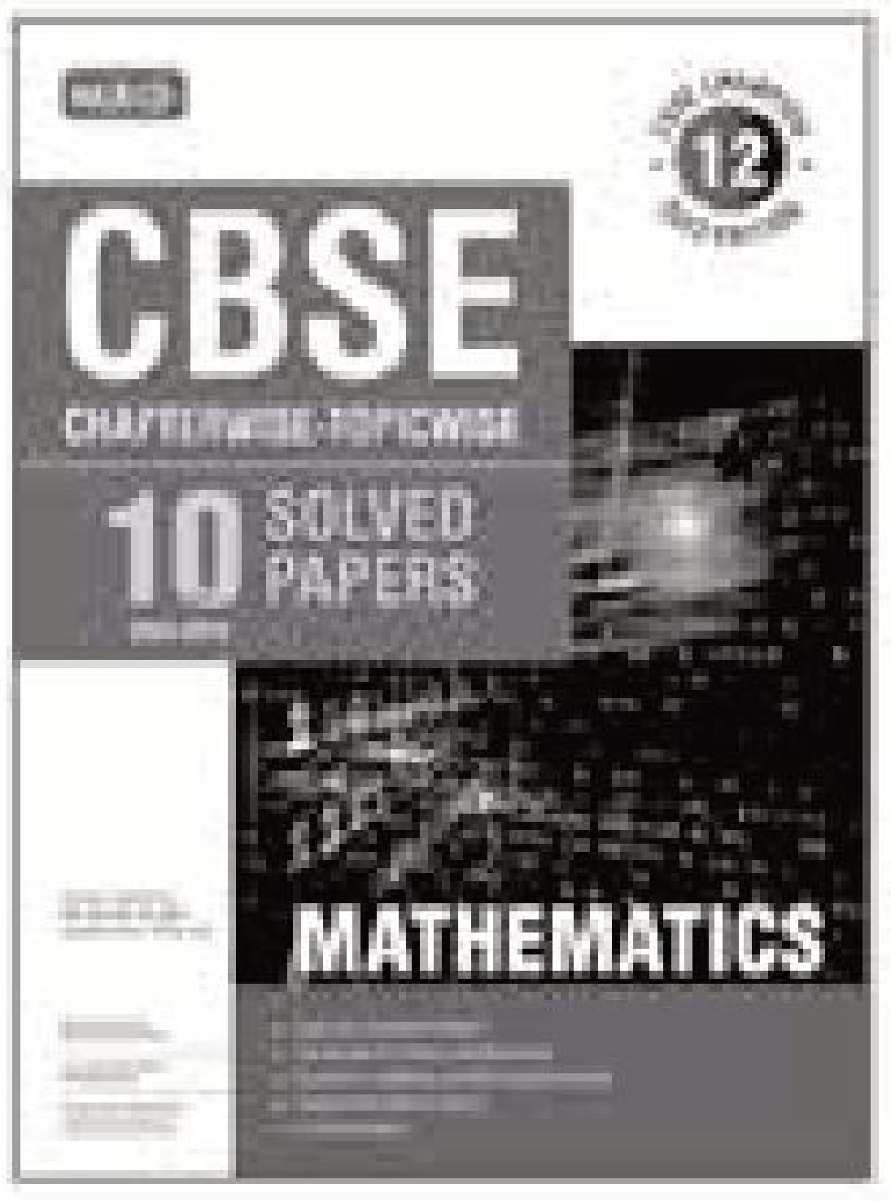
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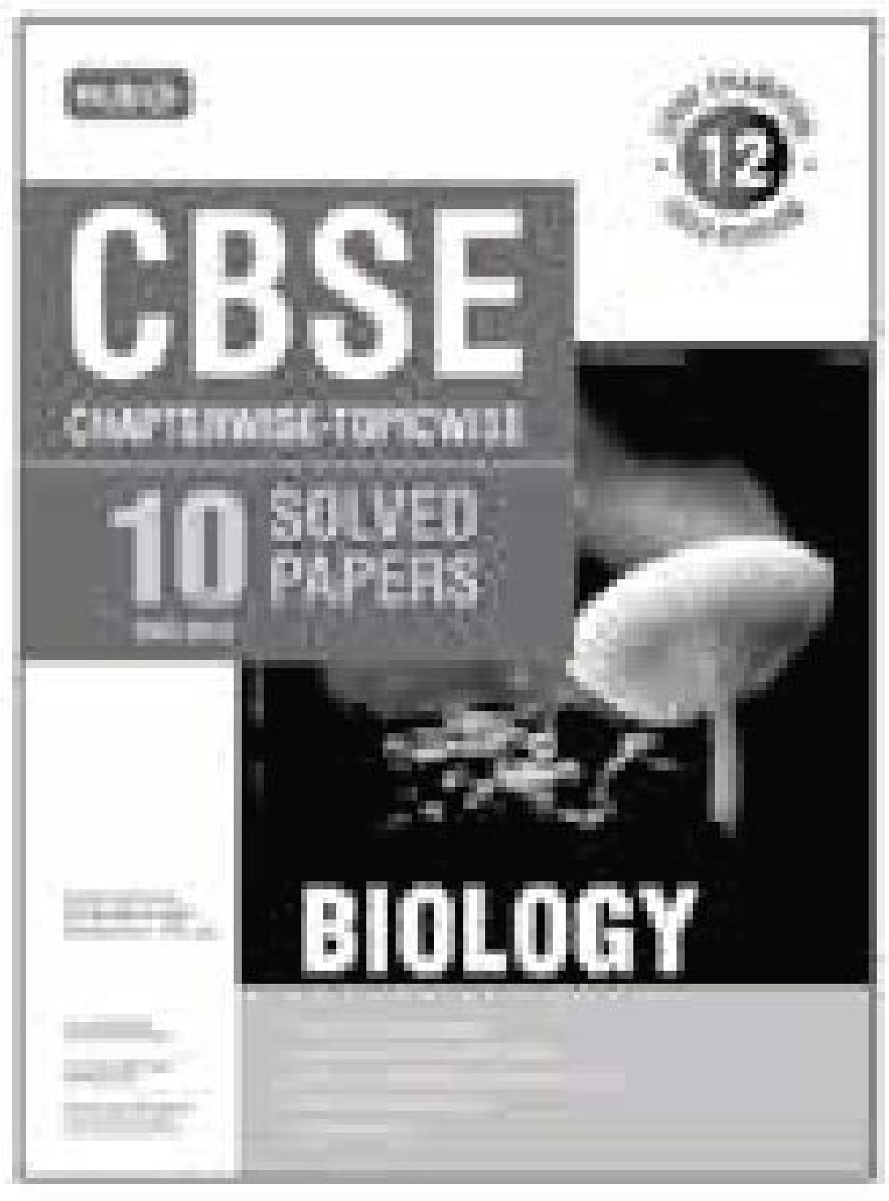
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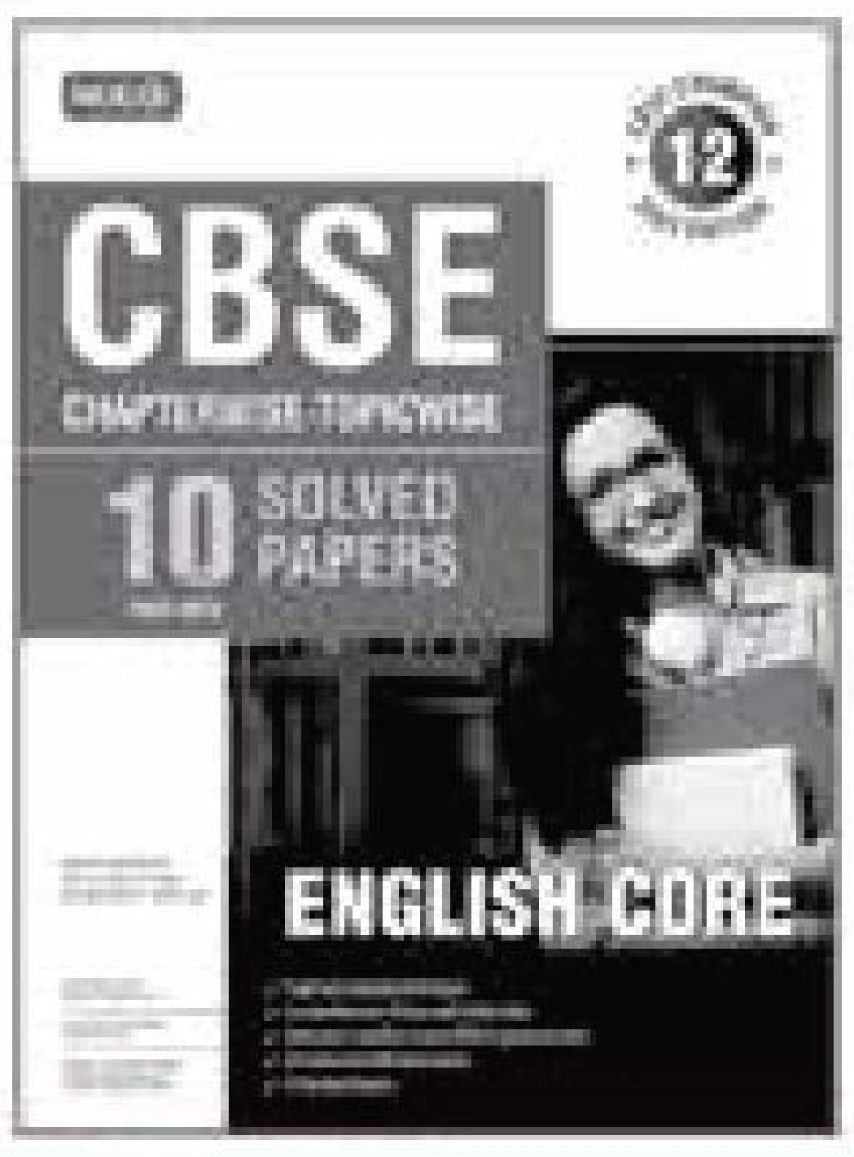
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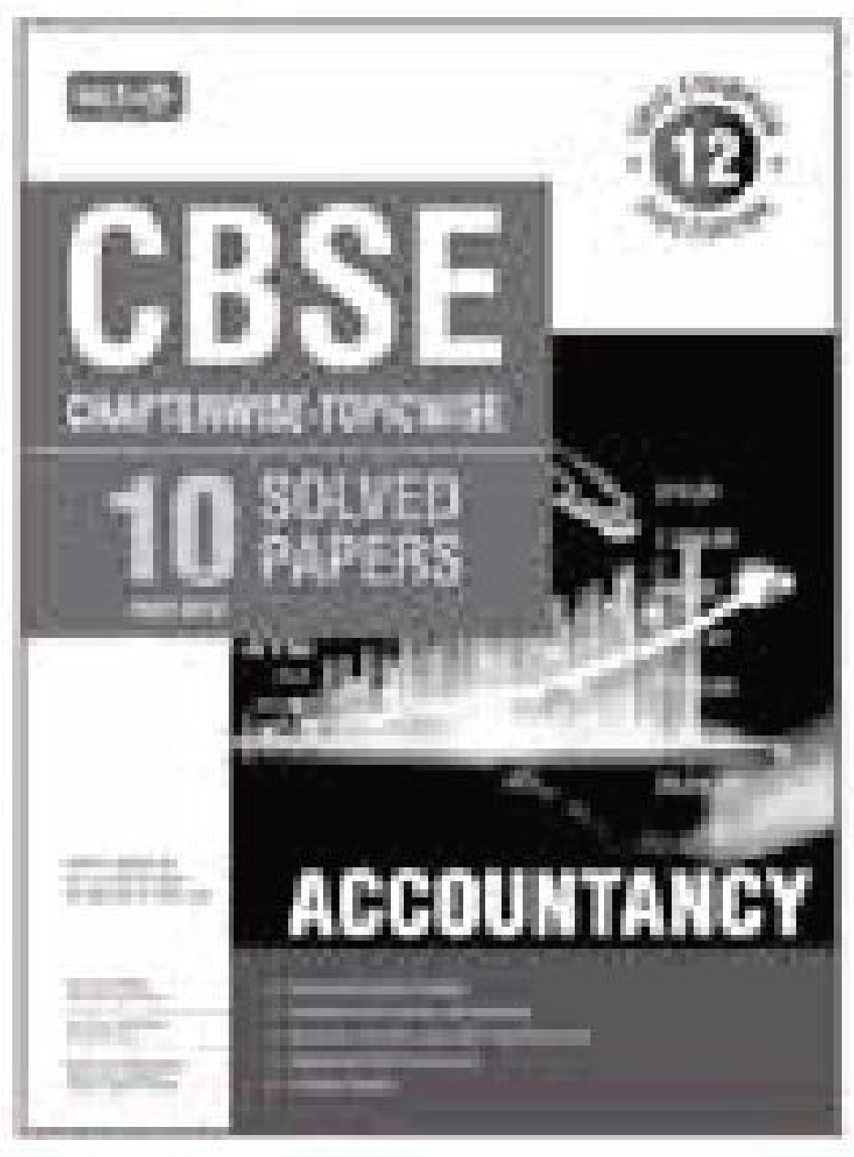
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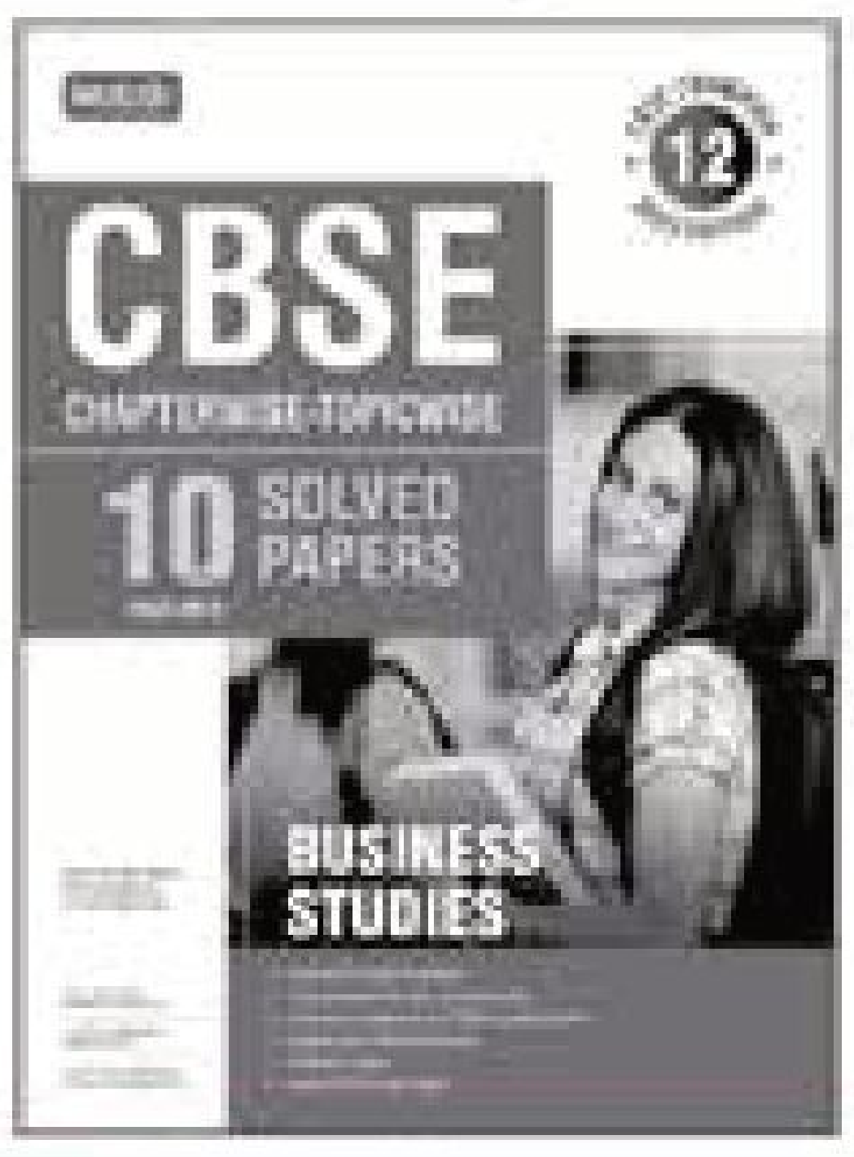
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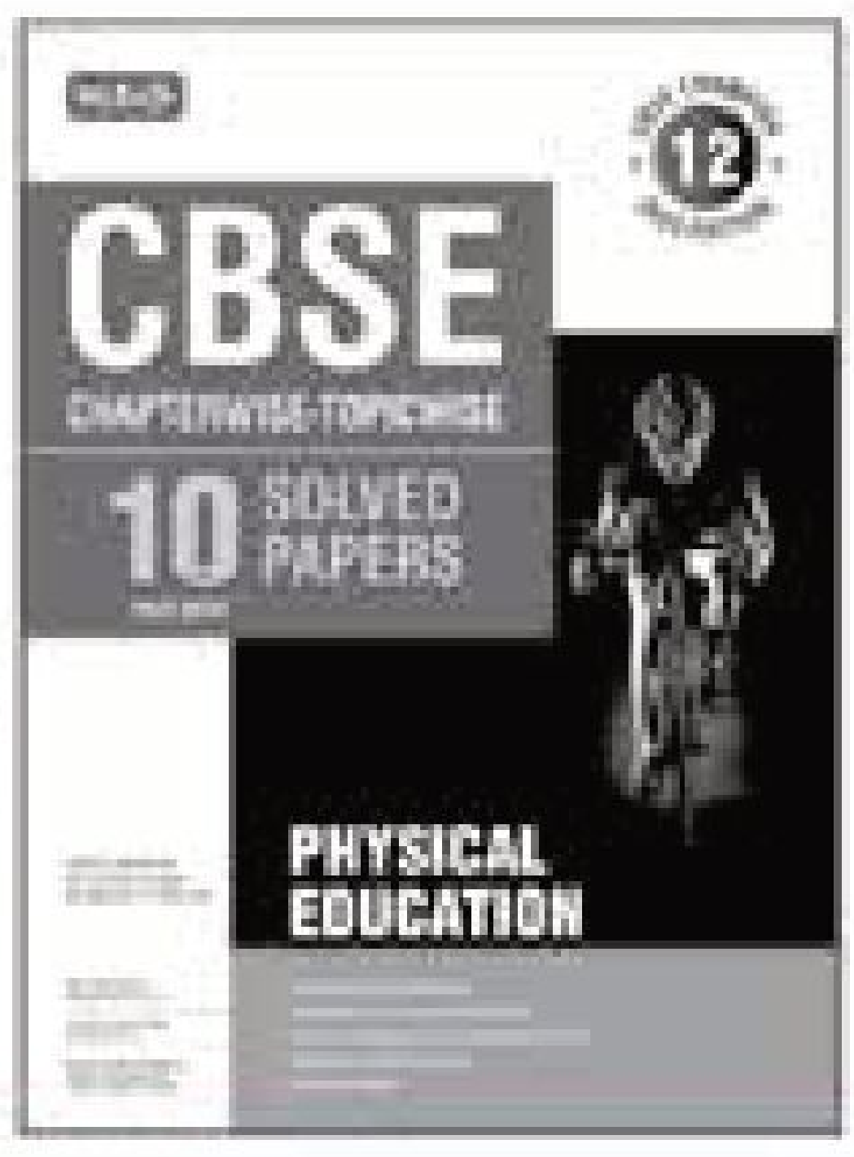
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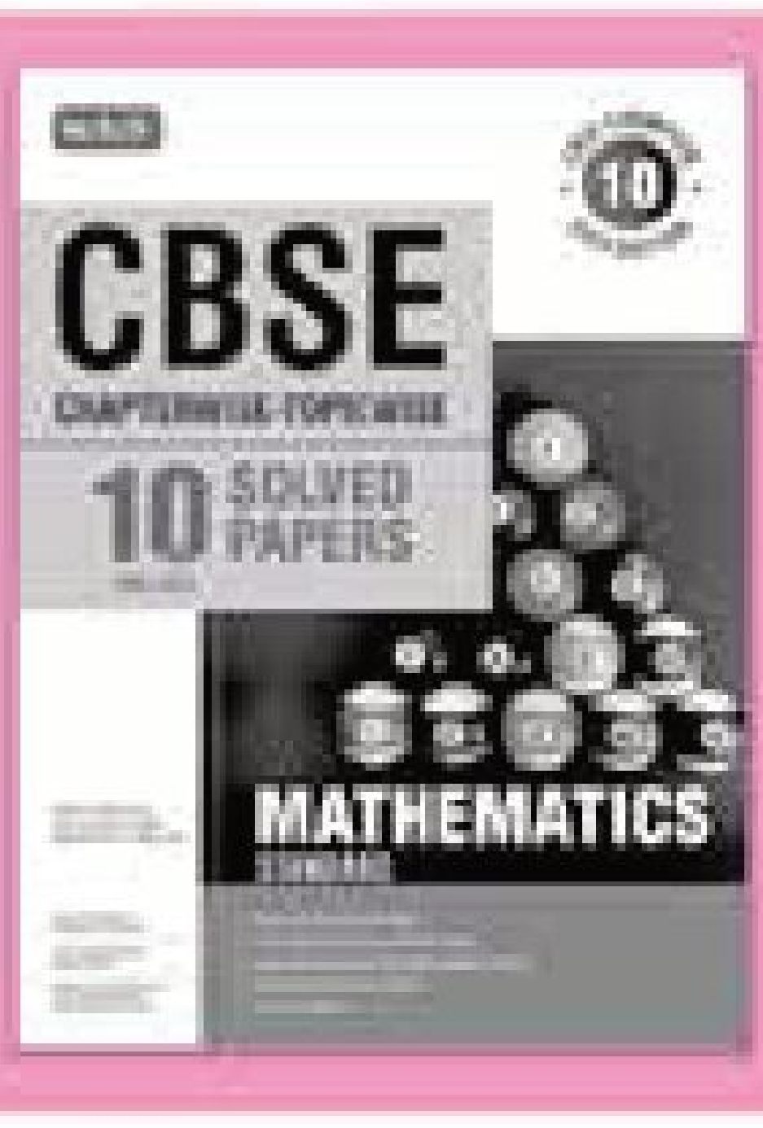
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**HIGHLIGHTS OF 2022-23 EDITION**

- Based on Reduced Syllabus
- Topicwise Graphical Analysis
- Quick Recap with Brain Map
- Previous 10 years (2022- 2013) questions of CBSE
- CBSE Cognitive Level Tagging
- Key Points, Answer Tips, Concept Applied, Shortcuts, Alternating Methods, Commonly Made Mistakes provided
- CBSE Topper's Answers
- Past 3 years CBSE Sample Question Papers
- Chapterwise Self-Assessment
- 5 Practice Papers

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$\Rightarrow\quad 4x^2(x-1) > 0 \Rightarrow x-1 > 0$ and $x \neq 0 \qquad [\because x^2 > 0]$

$\Rightarrow\quad x > 1 \Rightarrow x \in (1, \infty)$

For $f(x)$ to be decreasing, we must have $f'(x) < 0$

$\Rightarrow\quad 4x^2(x-1) < 0 \Rightarrow x-1 < 0$ and $x \neq 0$

$[\because x^2 > 0$ for all $x \neq 0]$

$\Rightarrow\quad x < 1$ and $x \neq 0 \Rightarrow x \in (-\infty, 0) \cup (0, 1)$

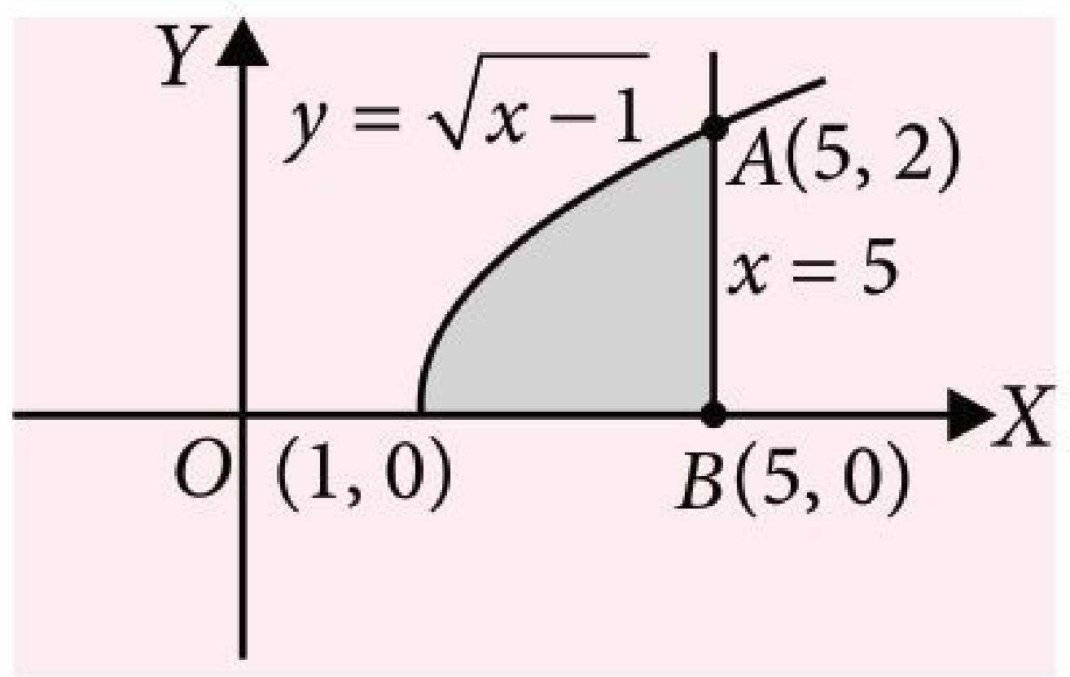
$\therefore\quad f$ is increasing in $[1, \infty)$ and decreasing in $(-\infty, 0) \cup (0, 1)$

**25.** We have, $y = \sqrt{x-1}$ ...(1)

and $x = 5$ ...(2)

Points of intersection are $O(1, 0)$, $A(5, 2)$

$\therefore$ Required area $= \int_1^5 \sqrt{x-1}\,dx$



$$= \left[\frac{2}{3}(x-1)^{3/2}\right]_1^5$$

$$= \frac{2}{3}[(4)^{3/2} - 0] = \frac{2}{3} \times 8$$

$$= \frac{16}{3} \text{ sq. units}$$

**26.** Given that, $a \geq 1$, $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$

$$\Rightarrow\quad f'(x) = \sqrt{3}\cos x - (-\sin x) - 2a$$

$$= \sqrt{3}\cos x + \sin x - 2a = 2\left[\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right] - 2a$$

$$= 2\left[\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x\right] - 2a = 2\cos\left(\frac{\pi}{6} - x\right) - 2a$$

$[\because \cos(A - B) = \cos A \cos B + \sin A \sin B]$

$$= 2\left[\cos\left(\frac{\pi}{6} - x\right) - a\right]$$

Since, $\cos x \in [-1, 1]$ and $a \geq 1$

So, $2\left[\cos\left(\frac{\pi}{6} - x\right) - a\right] \leq 0 \Rightarrow f'(x) \leq 0$

Therefore, $f(x)$ is a decreasing function in $R$.

Hence proved.

**27.** $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$\therefore\quad \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

Now, $\int \sin^3 x \cos\frac{x}{2}\,dx = \int \frac{3\sin x - \sin 3x}{4} \cdot \cos\frac{x}{2}\,dx$

$$= \frac{3}{4}\int \sin x \cos\frac{x}{2}\,dx - \frac{1}{4}\int \sin 3x \cos\frac{x}{2}\,dx$$

$$= \frac{3}{8}\int 2\sin x \cos\frac{x}{2}\,dx - \frac{1}{8}\int 2\sin 3x \cos\frac{x}{2}\,dx$$

$$= \frac{3}{8}\int\left(\sin\frac{3x}{2} + \sin\frac{x}{2}\right)dx - \frac{1}{8}\int\left(\sin\frac{7x}{2} + \sin\frac{5x}{2}\right)dx$$

$[\because 2\sin A \cos B = \sin(A + B) + \sin(A - B)]$

$$= \frac{3}{8}\int \sin\frac{3x}{2}\,dx + \frac{3}{8}\int \sin\frac{x}{2}\,dx - \frac{1}{8}\int \sin\frac{7x}{2}\,dx - \frac{1}{8}\int \sin\frac{5x}{2}\,dx.$$

$$= \frac{3}{8}\left(-\frac{\cos 3x/2}{\frac{3}{2}}\right) + \frac{3}{8}\cdot\left(-\frac{\cos x/2}{\frac{1}{2}}\right) - \frac{1}{8}\left(-\frac{\cos 7x/2}{\frac{7}{2}}\right) - \frac{1}{8}\left(-\frac{\cos 5x/2}{\frac{5}{2}}\right) + c$$

$$= -\frac{1}{4}\cos\frac{3x}{2} - \frac{3}{4}\cos\frac{x}{2} + \frac{1}{28}\cos\frac{7x}{2} + \frac{1}{20}\cos\frac{5x}{2} + c$$

**OR**

Let $I = \int \frac{3x-1}{\sqrt{x^2+9}}\,dx$ ; $I = \int \frac{3x}{\sqrt{x^2+9}}\,dx - \int \frac{1}{\sqrt{x^2+9}}\,dx$

$$= \int \frac{3x}{\sqrt{x^2+9}}\,dx - \int \frac{1\,dx}{\sqrt{x^2+(3)^2}}$$

$$\Rightarrow\quad I = I_1 - \log\left|x + \sqrt{x^2+9}\right| \qquad \text{...(i)}$$

where $I_1 = \int \frac{3x\,dx}{\sqrt{x^2+9}}$

Put $x^2 + 9 = t^2 \Rightarrow 2x\,dx = 2t\,dt \Rightarrow x\,dx = t\,dt$

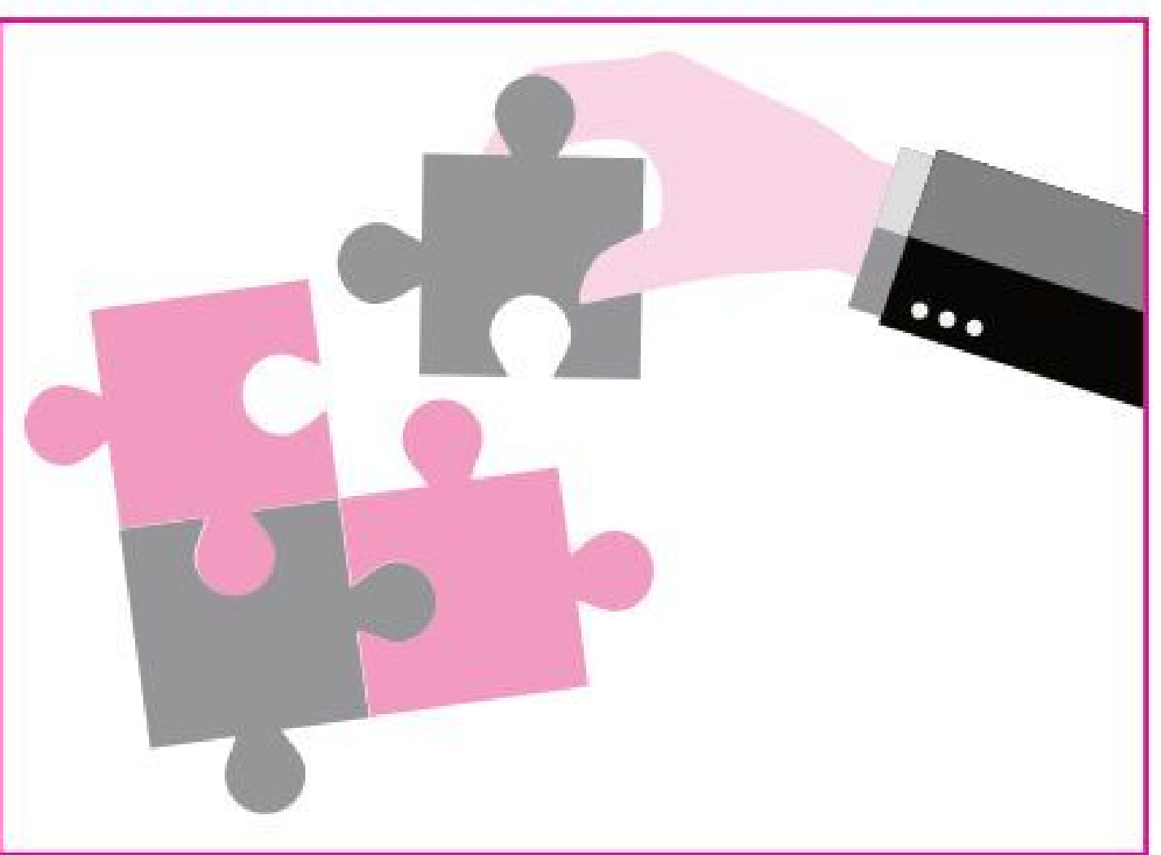
$$\therefore\quad I_1 = 3\int \frac{t}{t}\,dt = 3\int dt = 3t + C = 3\sqrt{x^2+9} + C$$

Substituting value of $I_1$ in (i), we get

$$I = 3\sqrt{x^2+9} - \log\left|x + \sqrt{x^2+9}\right| + C$$

## PUZZLE CORNER

ANSWER · OCTOBER 2022



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| 5 | 2÷<br>2 | 1 | 14+<br>6 | 4 | 3 |
| 2 | 1 | 2÷<br>3 | 4 | 6 | 5 |
| 1–<br>3 | 4 | 6 | 25×<br>5 | 2÷<br>2 | 1 |
| 10+<br>4 | 6 | 5 | 1 | 1–<br>3 | 2 |
| 1–<br>6 | 5 | 6×<br>2 | 3 | 3–<br>1 | 4 |

**28.** Let $I=\int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx$. Then,

$$I=\int_0^{\pi}\left\{\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{\sin(2\pi-x)}}\right\}dx$$

$$\left[\text{Using }\int_0^{2a}f(x)\,dx=\int_0^{a}(f(x)+f(2a-x))dx\right]$$

$$\Rightarrow I=\int_0^{\pi}\left\{\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{-\sin x}}\right\}dx$$

$$\Rightarrow I=\int_0^{\pi}\left\{\frac{1}{1+e^{\sin x}}+\frac{e^{\sin x}}{1+e^{\sin x}}\right\}dx=\int_0^{\pi}1dx=[x]_0^{\pi}=\pi$$

**OR**

Let $I=\int\frac{2x}{(x^2+1)(x^2+2)^2}dx$ . Put $x^2=y \Rightarrow 2xdx=dy$

$$\therefore \quad I=\int\frac{dy}{(y+1)(y+2)^2}$$

Let $\frac{1}{(y+1)(y+2)^2}=\frac{A}{y+1}+\frac{B}{y+2}+\frac{C}{(y+2)^2}$

$\Rightarrow \quad 1=A(y+2)^2+B(y+1)(y+2)+C(y+1)$ ...(i)

Putting $y=-1$ in (i), we get $1=A$

Putting $y=-2$ in (i), we get $1=-C \Rightarrow C=-1$

Putting $y=0$ in (i), we get $1=4A+2B+C$

$\Rightarrow \quad B=\frac{1-4+1}{2}=-1$

$$\therefore \quad I=\int\left[\frac{1}{y+1}-\frac{1}{y+2}-\frac{1}{(y+2)^2}\right]dy$$

$=\log|y+1|-\log|y+2|+\frac{1}{y+2}+c$

$=\log\left|\frac{y+1}{y+2}\right|+\frac{1}{y+2}+c \;=\log\left|\frac{x^2+1}{x^2+2}\right|+\frac{1}{x^2+2}+c$

$[\because y=x^2]$

**29.** Let $I=\int_0^{\infty}\frac{x^2}{(a^2+x^2)^{5/2}}dx$

Put $x=a\tan\theta \Rightarrow dx=a\sec^2\theta\, d\theta$.

When $x=0 \Rightarrow a\tan\theta=0 \Rightarrow \tan\theta=0; \Rightarrow \theta=0$

When $x=\infty \Rightarrow a\tan\theta=\infty \Rightarrow \tan\theta=\infty \Rightarrow \theta=\pi/2$

$$\therefore I=\int_0^{\pi/2}\frac{a^2\tan^2\theta}{(a^2+a^2\tan^2\theta)^{5/2}}a\sec^2\theta\, d\theta$$

$$=\frac{1}{a^2}\int_0^{\pi/2}\sin^2\theta\cos\theta\, d\theta$$

Put $t=\sin\theta \Rightarrow dt=\cos\theta\, d\theta$

When $\theta=0 \Rightarrow t=0$ ; When $\theta=\pi/2 \Rightarrow t=1$

$$\therefore I=\frac{1}{a^2}\int_0^1 t^2\,dt \;\Rightarrow I=\frac{1}{a^2}\left[\frac{t^3}{3}\right]_0^1=\frac{1}{3a^2}$$

**30.** Here, $f(x)=x^3+ax^2+bx+5\sin^2 x$

$\Rightarrow \quad f'(x)=3x^2+2ax+b+5\sin 2x>0$

$[\because f(x)$ is increasing$]$

Minimum value of $\sin 2x=-1$

$\therefore \quad 3x^2+2ax+b-5>0$

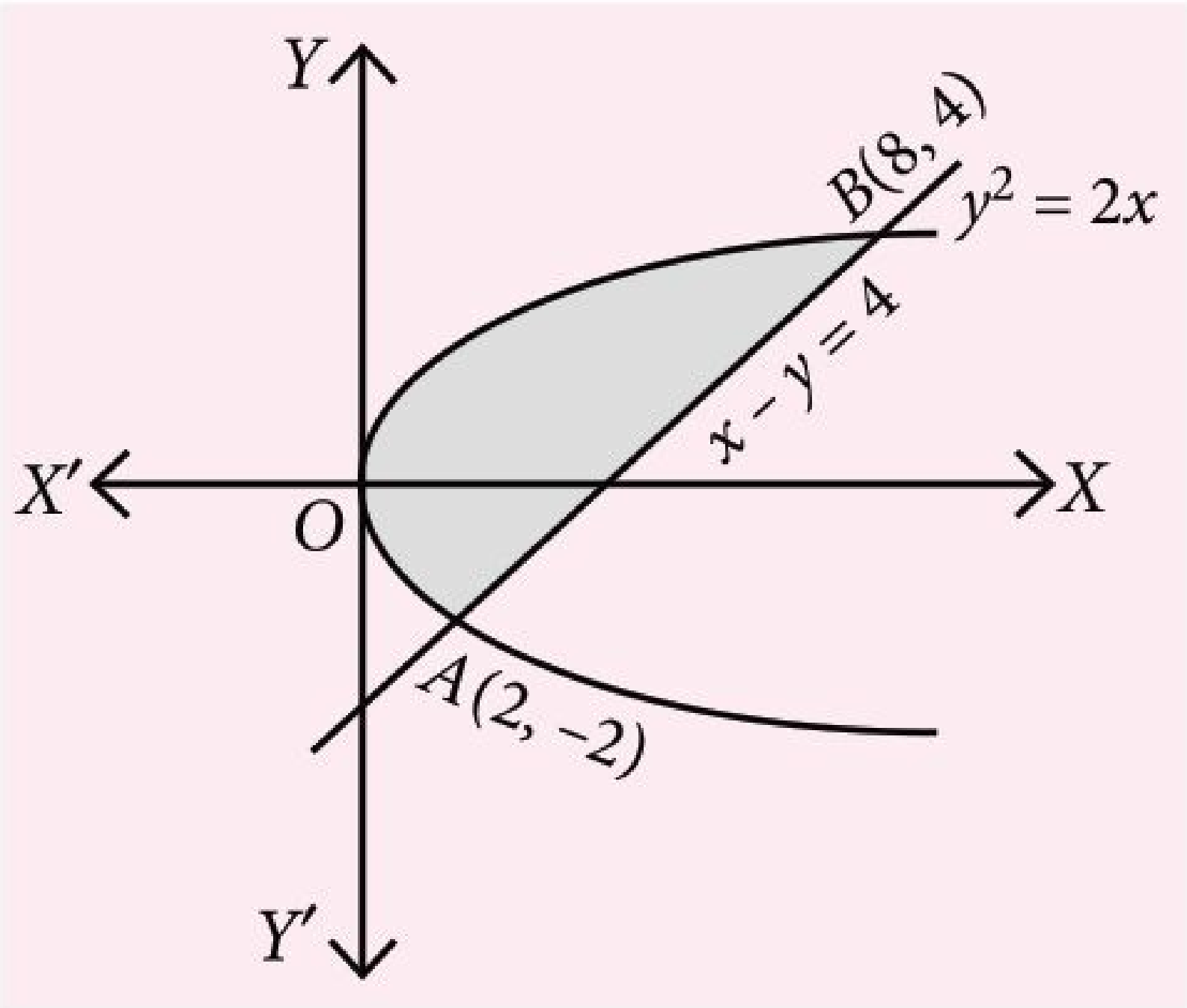
Since the above quadratic equation is > 0,

$\therefore$ discriminant should be < 0

$\therefore \quad (2a)^2-4(3)(b-5)<0$

$\Rightarrow \quad 4a^2-12b+60<0 \Rightarrow a^2-3b+15<0$

**31.** Given curves are $y^2=2x$ and $x-y=4$



Points of intersection are $A(2,-2)$ and $B(8,4)$

$$\therefore \quad \text{Required Area}=\int_{-2}^{4}(4+y)dy-\int_{-2}^{4}\frac{y^2}{2}dy$$

$$=\left[4y+\frac{y^2}{2}\right]_{-2}^{4}-\left[\frac{y^3}{6}\right]_{-2}^{4}$$

$$=16+\frac{16}{2}-\left(-8+\frac{4}{2}\right)-\left(\frac{64}{6}-\left(\frac{-8}{6}\right)\right)$$

$$=16+\frac{16}{2}+8-2-\frac{64}{6}-\frac{8}{6}=18\text{ sq. units}$$

**OR**

*Refer to answer 16, page no. 368-369 of MTG 100 percent mathematics, Class -12.*

**32.** Let $I=\int_0^1\frac{\log(1+x)}{1+x^2}dx$

Put $x=\tan\theta \Rightarrow dx=\sec^2\theta d\theta$

Also, when $x=0$, $\theta=0$ and when $x=1$, $\theta=\pi/4$

$$\therefore \quad I=\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{1+\tan^2\theta}\cdot\sec^2\theta d\theta=\int_0^{\pi/4}\log(1+\tan\theta)d\theta$$

$$=\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta=\int_0^{\pi/4}\log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta$$

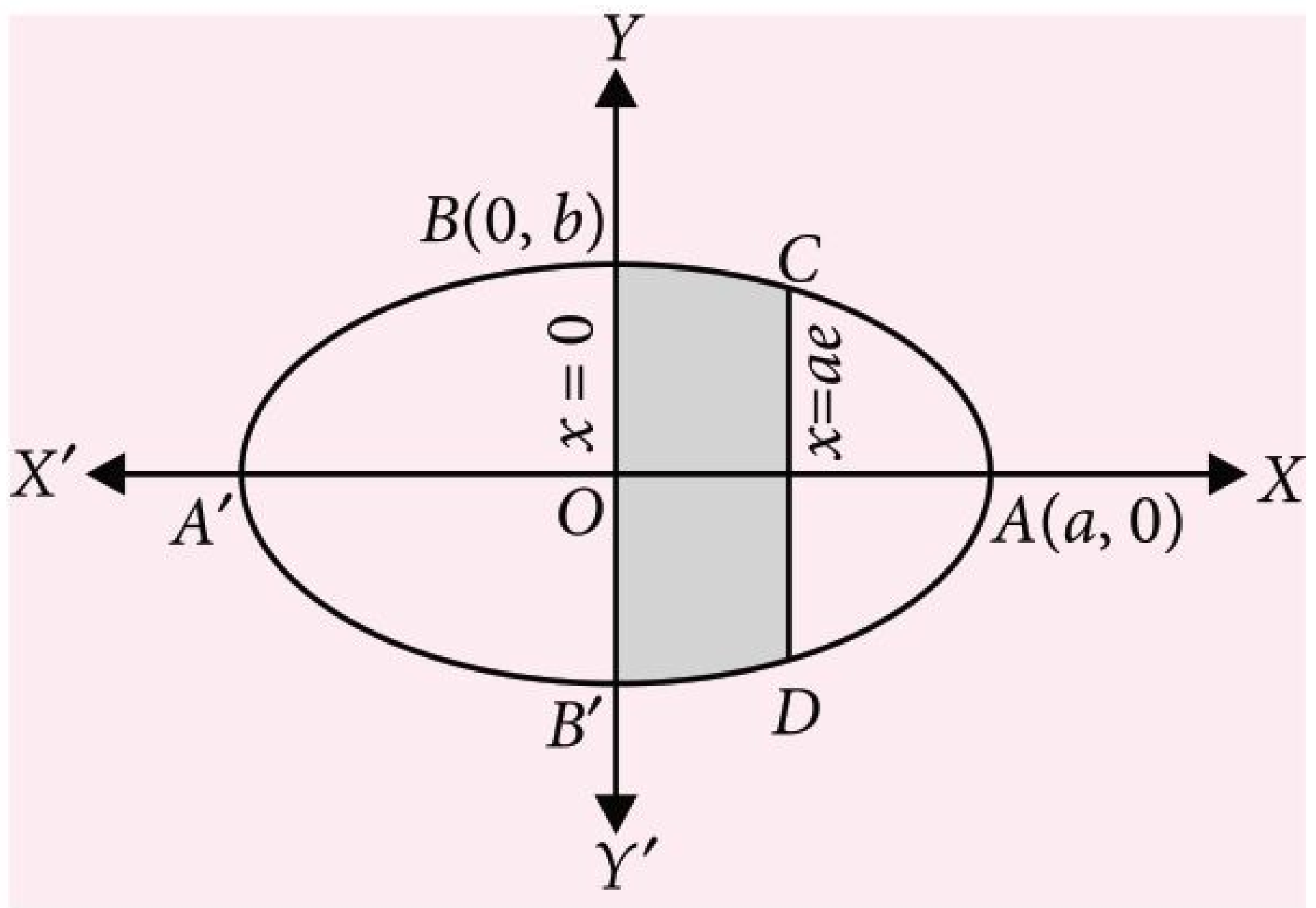
$$=\int_0^{\pi/4}[\log 2-\log(1+\tan\theta)]d\theta$$

$$=\log 2\int_0^{\pi/4}1\cdot d\theta-\int_0^{\pi/4}\log(1+\tan\theta)d\theta \;=\log 2[\theta]_0^{\pi/4}-I$$

$$\Rightarrow \quad 2I=\log 2\left[\frac{\pi}{4}-0\right]\Rightarrow I=\frac{\pi}{8}\log 2$$

**33.** The given ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

and line is $x = ae$



From (1), $\frac{y^2}{b^2}=1-\frac{x^2}{a^2} \Rightarrow y=\pm\frac{b}{a}\sqrt{a^2-x^2}$

$\therefore$ Required area = Shaded area $=2\int_0^{ae}\frac{b}{a}\sqrt{a^2-x^2}\,dx$

$$=\frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^{ae}$$

$$=\frac{2b}{a}\left[\left(\frac{ea}{2}\sqrt{a^2(1-e^2)}+\frac{a^2}{2}\sin^{-1}(e)\right)\right]$$

$$=abe\sqrt{1-e^2}+ab\sin^{-1}(e)$$

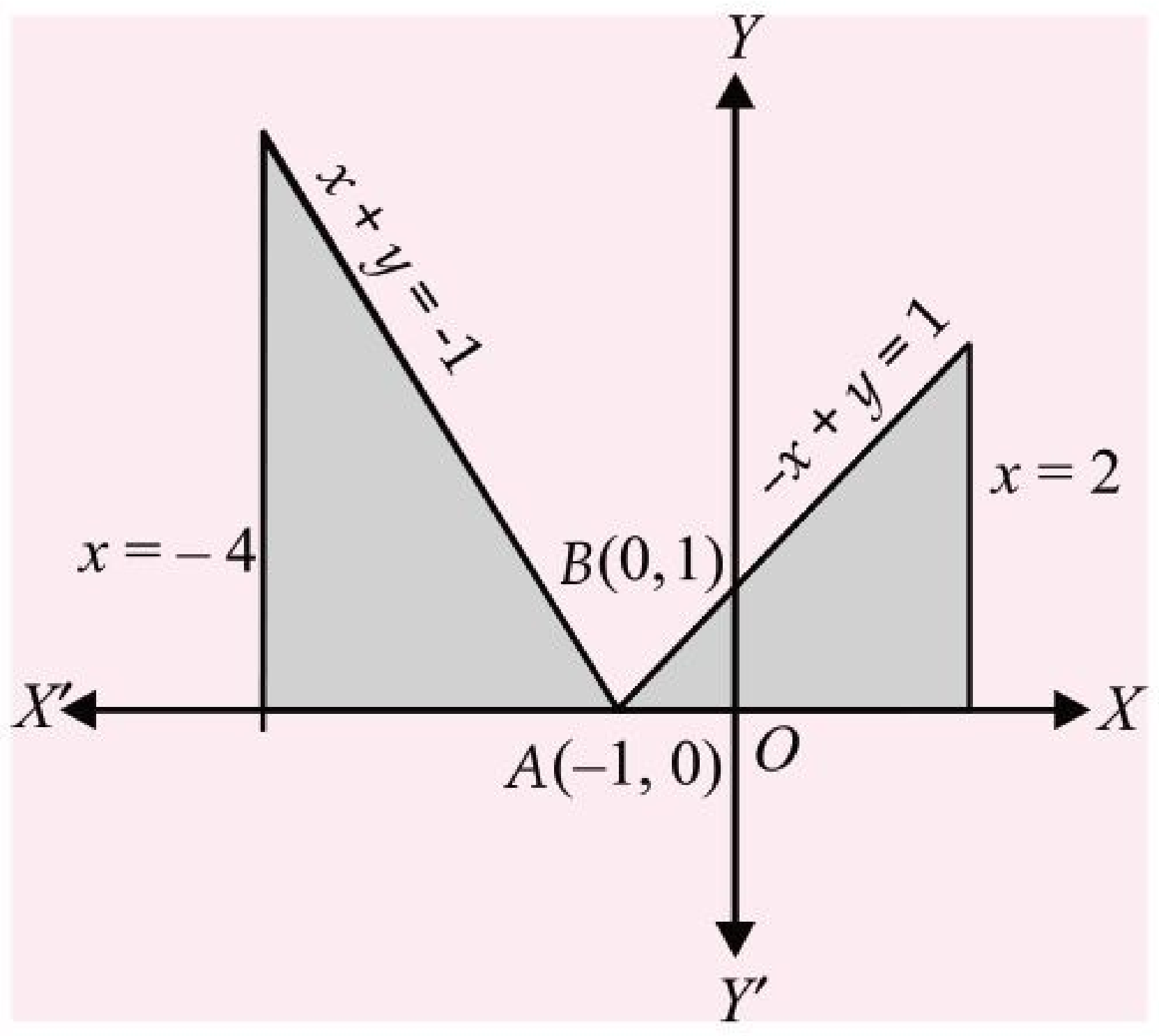
$$=ab[e\sqrt{1-e^2}+\sin^{-1}(e)]\,\text{sq. units}$$

**OR**

Here $y = |x+1| =\begin{cases} x+1, & \text{if } x\ge -1 \\ -x-1, & \text{if } x<-1\end{cases}$

Thus we get two lines $-x + y = 1$ ...(1), $x + y = -1$ ...(2)

Their graphs are as shown below and the region whose area to be calculated is shaded.



$\therefore$ Required area $=\int_{-4}^{2}|x+1|\,dx$

$$=\int_{-4}^{-1}(-x-1)dx+\int_{-1}^{2}(x+1)dx=-\left[\frac{x^2}{2}+x\right]_{-4}^{-1}+\left[\frac{x^2}{2}+x\right]_{-1}^{2}$$

$$=-\left[\left(\frac{1}{2}-1\right)-\left(\frac{16}{2}-4\right)\right]+\left[\left(\frac{4}{2}+2\right)-\left(\frac{1}{2}-1\right)\right]$$

$$=-\left[-\frac{1}{2}-4\right]+\left[4+\frac{1}{2}\right]=\frac{9}{2}+\frac{9}{2}=9\text{ sq. units}$$

**34.** We have, $f(x) = \sin 2x - x \Rightarrow f'(x) = 2\cos 2x - 1$

Now, $f'(x) = 0 \Rightarrow 2\cos 2x - 1 = 0$

$$\Rightarrow \cos 2x=\frac{1}{2}\Rightarrow 2x=\frac{-\pi}{3}\text{ or }2x=\frac{\pi}{3}\qquad\left[\because \frac{-\pi}{2}\le x\le\frac{\pi}{2}\right]$$

$\therefore$ $x=\frac{-\pi}{6}$ and $x=\frac{\pi}{6}$ are the critical points.

$f''(x) = -4\sin 2x$

At $x=\frac{\pi}{6}$, $f''\left(\frac{\pi}{6}\right)=-4\sin\frac{2\pi}{6}=\frac{-4\sqrt{3}}{2}<0.$

So, $x=\frac{\pi}{6}$ is a point of local maximum.

The local maximum value is

$$f\left(\frac{\pi}{6}\right)=\sin\left(\frac{2\cdot\pi}{6}\right)-\frac{\pi}{6}=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$$

At $x=\frac{-\pi}{6}$, $f''\left(\frac{-\pi}{6}\right)=-4\sin\left(\frac{-\pi}{3}\right)=2\sqrt{3}>0$

So, $x=-\frac{\pi}{6}$ is a point of local minimum.

The local minimum value is

$$f\left(\frac{-\pi}{6}\right)=\sin 2\left(\frac{-\pi}{6}\right)+\frac{\pi}{6}=\frac{-\sqrt{3}}{2}+\frac{\pi}{6}$$

**OR**

Let $AR$ be $x$ m. Then, $RB = (20 - x)$ m.

In $\Delta RAP$ and $\Delta RBQ$, we have

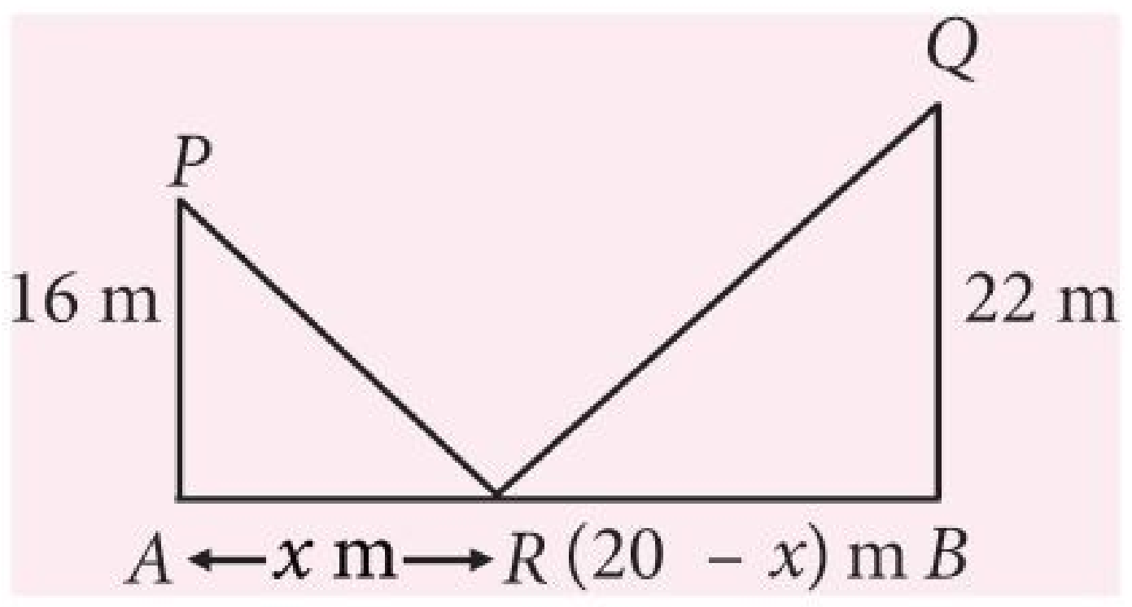
$PR^2 = x^2 + 16^2$ and $RQ^2 = 22^2 + (20 - x)^2$

$\therefore$ $PR^2 + RQ^2 = x^2 + 16^2 + 22^2 + (20 - x)^2$

$= 2x^2 - 40x + 1140$

Let $Z = PR^2 + RQ^2$.

Then, $Z = 2x^2 - 40x + 1140$.



Clearly, $Z$ will be maximum or minimum according as $PR^2 + RQ^2$ will be maximum or minimum.

Now, $Z = 2x^2 - 40x + 1140$

$\Rightarrow \frac{dZ}{dx}=4x-40$

For maximum or minimum, we must have $\frac{dZ}{dx}=0$

$\Rightarrow$ $4x - 40 = 0 \Rightarrow x = 10$

Clearly, $\frac{d^2Z}{dx^2}=4>0$ for all $x$

$\therefore$ $Z$ is minimum when $x = 10$.

Thus, $RP^2 + RQ^2$ is minimum when distance of $R$ from $A$ is 10 m.

**35.** *Refer to answer 15, page no. 317 of MTG 100 percent mathematics, Class -12.*

**36. (i)** Let $I = \int \frac{(x+1)^2}{x(x^2+1)}dx = \int \frac{x^2+1+2x}{x(x^2+1)}dx$

$= \int\left(\frac{1}{x}+\frac{2}{x^2+1}\right)dx = \log|x| + 2\tan^{-1}x + c$

**(ii)** $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}dx$

$$=\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c=\frac{x}{2}-\frac{\sin 2x}{4}+c$$

**(iii)** Let $I_1 = \int \tan^2 x\,dx = \int(\sec^2 x - 1)\,dx = \tan x - x + c$

Let $I_2 = \int \frac{dx}{\sin^2 x\cos^2 x} = \int \frac{4}{4\sin^2 x\cos^2 x}dx$

$= 4\int \operatorname{cosec}^2 2x\,dx = -2\cot 2x + c$

$\therefore$ $I = \tan x - x - 2\cot 2x + c$

**OR**

Let $I = \int e^x\left(\frac{x+2}{x+4}\right)^2 dx = \int e^x\left(\frac{x+4-2}{x+4}\right)^2 dx$

$= \int e^x\left(1-\frac{2}{x+4}\right)^2 dx = \int e^x\left[\left(1-\frac{4}{x+4}\right)+\frac{4}{(x+4)^2}\right]dx$

$= \int e^x\left[\left(1-\frac{4}{(x+4)}\right)+\frac{d}{dx}\left(1-\frac{4}{(x+4)}\right)\right]dx$

$= \int e^x\left(1-\frac{4}{x+4}\right)+C$

$\left[\because \int e^x[f(x)+f'(x)]dx = e^x f(x)+C\right]$

**37. (i)** We have, $x^2 + y^2 = 4$ ...(1)

and $x = \sqrt{3}y$ ...(2)

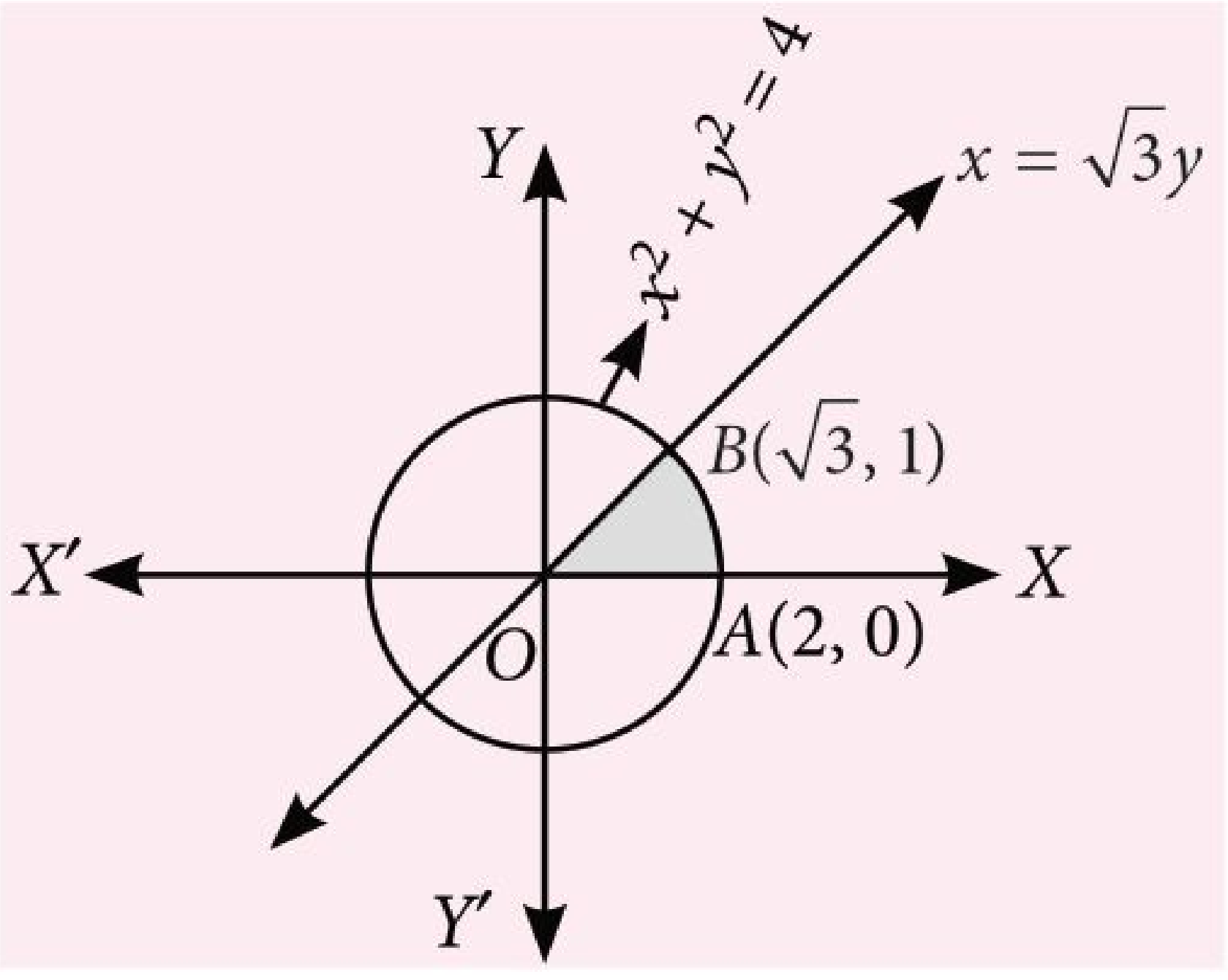
From (1) and (2), we get

$3y^2 + y^2 = 4 \Rightarrow 4y^2 = 4 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

From (2), $x = \sqrt{3}, -\sqrt{3}$

$\therefore$ Points of intersection of pizza and edge of knife are $(\sqrt{3}, 1), (-\sqrt{3}, -1)$.

**(ii)**



**(iii)** Required area $= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}dx + \int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$

$$=\frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}}+\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^2$$

$$=\frac{1}{\sqrt{3}}\left[\frac{3}{2}-0\right]+\left[2\sin^{-1}(1)-\left(\frac{\sqrt{3}}{2}+2\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$=\frac{\sqrt{3}}{2}+\frac{2\pi}{2}-\frac{\sqrt{3}}{2}-\frac{2\pi}{3}=\frac{\pi}{3}\text{ sq. units}$$

**OR**

We have, $x^2 + y^2 = 4$

$\Rightarrow$ $(x-0)^2 + (y-0)^2 = (2)^2$

$\therefore$ Radius = 2

Area of $\frac{1}{4}$th slice of pizza $= \frac{1}{4}\pi(2)^2 = \pi$ sq. units

Differentiate $x^2 + y^2 = 4$ w.r.t $x$.

$2x+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{-x}{y}$ $\therefore \left(\frac{dy}{dx}\right)_{(\sqrt{3},1)} = -\sqrt{3}$.

**38. (i)** Given, perimeter of window = 10 m

$\therefore x+2y+\pi\frac{x}{2}=10$. Now, $\Rightarrow y=5-\frac{x}{2}-\frac{\pi x}{4}$

$A = x\cdot y + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$=x\left(5-\frac{x}{2}-\frac{\pi x}{4}\right)+\frac{1}{2}\frac{\pi x^2}{4}$ $\left[\because \text{From (i), } y=5-\frac{x}{2}-\frac{\pi x}{4}\right]$

$\frac{dt}{dx}=0\Rightarrow\frac{20}{\pi+4}$ and $\frac{d^2t}{dx}<0$ at $x=\frac{20}{p+4}$

$$=5x-\frac{x^2}{2}-\frac{\pi x^2}{4}+\frac{\pi x^2}{8}=5x-\frac{x^2}{2}-\frac{\pi x^2}{8}$$

**(ii)** At $x = \frac{20}{4+\pi}$

$$A=5\left(\frac{20}{4+\pi}\right)-\left(\frac{20}{4+\pi}\right)^2\frac{1}{2}-\frac{\pi}{8}\left(\frac{20}{4+\pi}\right)^2$$

$$=\frac{100}{4+\pi}-\frac{200}{(4+\pi)^2}-\frac{50\pi}{(4+\pi)^2}$$

$$=\frac{200+50\pi}{(4+\pi)^2}=\frac{50(4+\pi)}{(4+\pi)^2}=\frac{50}{4+\pi}$$

Now, We have, $y=5-\frac{x}{2}-\frac{\pi x}{4}=5-x\left(\frac{1}{2}+\frac{\pi}{4}\right)$

$$=5-x\left(\frac{2+\pi}{4}\right)=5-\left(\frac{20}{4+\pi}\right)\left(\frac{2+\pi}{4}\right)$$

$$=5-5\frac{(2+\pi)}{4+\pi}=\frac{20+5\pi-10-5\pi}{4+\pi}=\frac{10}{4+\pi}$$

◆

# MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

**Total Marks : 80**

## Series 4 : Continuity and Differentiability

**Time Taken : 60 Min.**

### Only One Option Correct Type

**1.** Let $f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}$, then the points of discontinuity of $g(x) = f[f(x)]$ is/are

(a) only $x = 1$ (b) $x = 2, 3$
(c) only $x = 3$ (d) $x = 1, 2$

**2.** Let $f(x)=\begin{cases}\left[\tan\left(\dfrac{\pi}{4}+x\right)\right]^{1/x}, & x\ne 0\\ k, & x=0\end{cases}$. For what value of $k$ $f(x)$ is continuous at $x = 0$?

(a) 1 (b) $e$ (c) $\frac{1}{e}$ (d) $e^2$

**3.** Number of points of discontinuity of $f(x) = [2x^3 - 5]$ in $[1, 2)$, is equal to (where $[x]$ denotes greatest integer less than or equal to $x$)

(a) 14 (b) 13 (c) 10 (d) 8

**4.** Let $f$ be a differentiable function satisfying the condition $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ for all $x, y\ (\ne 0) \in R$ and $f(y) \ne 0$. If $f'(1) = 2$, then $f'(x)$ is equal to

(a) $2f(x)$ (b) $\frac{f(x)}{x}$ (c) $2x\,f(x)$ (d) $\frac{2f(x)}{x}$

**5.** If $f(x) = (1 + x)(3 + x^2)^{1/2}\,(9 + x^3)^{1/3}$, then $f'(-1)$ is equal to

(a) 0 (b) $2\sqrt{2}$ (c) 4 (d) 6

**6.** If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then $f'(\pi/4)$ equals

(a) $-\sqrt{2}$ (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

### One or More Than One Option(s) Correct Type

**7.** If $y=e^{\sqrt{x}}+e^{-\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to

(a) $\dfrac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2\sqrt{x}}$ (b) $\dfrac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2x}$

(c) $\dfrac{1}{2\sqrt{x}}\sqrt{y^2-4}$ (d) $\dfrac{1}{2\sqrt{x}}\sqrt{y^2+4}$

**8.** If $y=x^{(\log x)^{\log(\log x)}}$, then $\frac{dy}{dx}$ is

(a) $\dfrac{y}{x}(\log x^{\log x-1})+2\log x\log(\log x)$

(b) $\dfrac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x)+1)$

(c) $(\log x)^2 + 2\log(\log x)$

(d) $\dfrac{y\log y}{x\log x}(2\log(\log x)+1)$

**9.** Let $f(x) = \min\{x^3, x^4\}$ for all $x \in R$. Then

(a) $f(x)$ is continuous for all $x$
(b) $f(x)$ is differentiable for all $x$
(c) $f'(x) = 3x^2$ for all $x > 1$
(d) $f(x)$ is not differentiable at two points.

**10.** If $f(x)=\begin{cases}-x-\dfrac{\pi}{2}, & x\le -\dfrac{\pi}{2}\\ -\cos x, & -\dfrac{\pi}{2}<x\le 0\\ x-1, & 0<x\le 1\\ \ln x, & x>1,\end{cases}$ then

(a) $f(x)$ is continuous at $x=-\frac{\pi}{2}$
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x)$ is differentiable at $x = 1$
(d) $f(x)$ is differentiable at $x=-\frac{3}{2}$

**11.** If $f(x)=\begin{cases}\dfrac{e^{ax}-1}{1-e^{bx}}, & x\neq 0\\ \dfrac{-a}{b}, & x=0\end{cases}$, then

(a) $f(x)$ is continuous at $x = 0$
(b) $f(x)$ is differentiable at $x = 0$
(c) $f'(0)=\dfrac{a}{2b}(b-a)$
(d) $f(x)$ is not differentiable at $x = 0$

**12.** If $f(x) = |x^2 - 3|x| + 2|$, then which of the following is/are true?

(a) $f'(x) = 2x - 3$ for $x \in (0, 1) \cup (2, \infty)$
(b) $f'(x) = 2x + 3$ for $x \in (-\infty, -2) \cup (-1, 0)$
(c) $f'(x) = -2x - 3$ for $x \in (-2, -1)$
(d) none of these

## Comprehension Type

### Paragraph for Q. No. 13 to 15

Consider $\dfrac{df(x)}{dg(x)}=\dfrac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}=\dfrac{f'(x)}{g'(x)}.$

**13.** The differentiation of $\sin^3 x + \cos^3 x$ with respect to $\sin x + \cos x$ is

(a) $\sin x \cos x$ (b) $-3 \sin x \cos x$
(c) $-\sin x \cos x$ (d) $3 \sin^2 x \cos^2 x$

**14.** The differentiation of $\tan^{-1}\dfrac{3x-x^3}{1-3x^2}$ with respect to $\tan^{-1}\dfrac{2x}{1-x^2}$, where $x\in\left(-\dfrac{1}{\sqrt{3}},\dfrac{1}{\sqrt{3}}\right)$ is

(a) 2 (b) 3 (c) 2/3 (d) 3/2

**15.** The differentiation of $\tan^{-1}\dfrac{2x}{1-x^2}$ with respect to $\sin^{-1}\dfrac{2x}{1+x^2}$, where $x \in (-1, 1)$ is

(a) 1 (b) $-1$ (c) 2 (d) $-2$

## Matrix Match Type

**16.** Match the following :

| | Column I | | Column II |
|---|---|---|---|
| (P) | $f(x) = x^2 \sin(1/x)$, $f(0) = 0$, then | (1) | $f$ is continuous but not derivable at $x = 0$ |
| (Q) | $f(x)=\dfrac{(x-4x^4)^{1/2}-(x/4)^{1/3}}{1-(8x^3)^{1/4}}$, $f(1/2) = 2/9$, then | (2) | $f$ is differentiable every where but $f'$ is not continuous |
| (R) | $f(x) = x \sin 1/x$, $f(0) = 0$, then | (3) | $f$ is not continuous at $x=\dfrac{1}{2}$ |
| (S) | $f(x) = x^3 \sin 1/x$, $f'(0) = 0$, then | (4) | $f'$ is continuous but not derivable at $x = 0$ |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 1 | 2 | 3 | 4 |
| (b) | 2 | 1 | 3 | 4 |
| (c) | 2 | 3 | 1 | 4 |
| (d) | 2 | 3 | 4 | 1 |

## Numerical Answer Type

**17.** The value of $f(0)$ for which $f(x)=\dfrac{512(\sqrt{x+4}-2)}{\sin 2x}$ is continuous, is ______.

**18.** $f(x)=\begin{cases}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right)^{1/x}, & x\neq 0\\ k, & x=0\end{cases}$. Then $f(x)$ is continuous at $x = 0$ if $k = e^m$. Find the value of $m$.

**19.** Number of real roots of the equation $e^{x-8} + 2x - 17 = 0$ is ______.

**20.** If $y=2\tan^{-1}\left(\dfrac{x\sqrt{2}}{1-x^2}\right)+\log\left(\dfrac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2}\right)$, then $\dfrac{dy}{dx}=\dfrac{k\sqrt{2}}{1+x^4}$, where $k$ is equal to ______.

⁂

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | **EXCELLENT WORK !** | You are well prepared to take the challenge of final exam. |
| 90-75% | **GOOD WORK !** | You can score good in the final exam. |
| 74-60% | **SATISFACTORY !** | You need to score more next time. |
| < 60% | **NOT SATISFACTORY!** | Revise thoroughly and strengthen your concepts. |

**Based on CBSE Circular released on 16th Sept. 2022**

# CBSE warm-up!

**CLASS-XI**

**Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.**

## Series-4

## Sequences and Series | Straight Lines

Time Allowed : 3 hours
Maximum Marks : 80

***General Instructions :***

- *(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
- *(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
- *(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
- *(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
- *(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
- *(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

### SECTION - A (MULTIPLE CHOICE QUESTIONS)

**Each question carries 1 mark.**

**1.** In an A.P., if $m^{\text{th}}$ term is $n$ and the $n^{\text{th}}$ term is $m$, where $m \neq n$, then find its $p^{\text{th}}$ term.

(a) $n - m + p$ (b) $n + m + p$
(c) $n + m - p$ (d) $n - m - p$

**2.** The line through the points $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through the points $(8, 12)$ and $(x, 24)$. Find the value of $x$.

(a) 1 (b) 2 (c) 3 (d) 4

**3.** If the sum of $n$ terms of an A.P. is $nP + \frac{1}{2}n(n-1)Q$, where $P$ and $Q$ are constants, find the common difference.

(a) $P + Q$ (b) $P$ (c) $P - Q$ (d) $Q$

**4.** Find the angle between the $x$-axis and the line joining the points $(3, -1)$ and $(4, -2)$.

(a) 130° (b) 135°
(c) 150° (d) None of these

**5.** Let the positive numbers $a, b, c, d$ be in A.P. Then $\frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ are in

(a) A.P. (b) G.P.
(c) A.G.P. (d) none of these

**6.** Find the equations of the lines parallel to axes and passing through $(-2, 3)$.

(a) $x = -2, y = 3$ (b) $x = 2, y = -3$
(c) $x = 2, y = 3$ (d) None of these

**7.** The income of a person is ₹ 300000, in the first year and he receives an increase of ₹ 10000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

(a) ₹ 7800000 (b) ₹ 7900000
(c) ₹ 7700000 (d) ₹ 790000

**8.** Find the equation of a line which passes through the point $(2, 3)$ and makes an angle of 30° with the positive direction of $x$-axis.

(a) $x-\sqrt{3}y+(3\sqrt{3}-2)=0$

(b) $x+\sqrt{3}y+(3\sqrt{3}-2)=0$

(c) $x-\sqrt{3}y=0$

(d) $x+\sqrt{3}y=0$

9. The 6th term of an A.P. is 18 and the 9th term is 12. The 15th term is equal to
(a) 0 (b) 1 (c) 2 (d) 3

10. The inclination of the line $x - y + 3 = 0$ with the positive direction of $x$-axis is
(a) 45° (b) 135° (c) −45° (d) −135°

11. Find the 10th and $n$th terms of the G.P. 5, 25, 125, …
(a) $7^{10}, 7^n$ (b) $4^{10}, 4^n$ (c) $5^{10}, 5^n$ (d) $6^{10}, 6^n$

12. Find the distance of the point (3, −5) from the line $3x - 4y - 26 = 0$.
(a) $\frac{2}{5}$ unit (b) 4 units (c) $\frac{3}{5}$ unit (d) 6 units

13. Which term of the G.P., 2, 8, 32, …, upto $n$ terms is 131072?
(a) 9th (b) 7th (c) 8th (d) 11th

14. The distance of the point $P(1, -3)$ from the line $2y - 3x = 4$ is
(a) 13 units (b) $\frac{7}{13}\sqrt{13}$ units
(c) $\sqrt{13}$ units (d) $13\sqrt{7}$ units

15. The minimum value of expression $3^x + 3^{1-x}, x \in R$, is
(a) 0 (b) $\frac{1}{3}$ (c) 3 (d) $2\sqrt{3}$

16. Find the angle between the lines joining the points (0, 0), (2, 3) and the points (2,−2), (3,5).
(a) $\tan^{-1}\left(\frac{11}{24}\right)$ (b) $\tan^{-1}\left(\frac{11}{23}\right)$
(c) $\cot^{-1}\left(\frac{11}{23}\right)$ (d) None of these

17. The ratio of the sum of first 3 terms to the sum of first 6 terms of a G.P. is 125 : 152. The common ratio of the G.P., is
(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

18. If the slope of the line joining the points (3, 4) and (− 2, $a$) is equal to $-\frac{2}{5}$, then the value of $a$ is equal to
(a) 6 (b) 4 (c) 3 (d) 2

## ASSERTION-REASON BASED QUESTIONS

**In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.**

**(a) Both A and R are true and R is the correct explanation of A.**

**(b) Both A and R are true but R is not the correct explanation of A.**

**(c) A is true but R is false.**

**(d) A is false but R is true.**

19. **Assertion (A) :** If $AB + BC = AC$, then $\Delta\ ABC$ can not be possible.
**Reason (R) :** Triangle $ABC$ will be possible only if $A$, $B$ and $C$ are non collinear.

20. **Assertion (A) :** $S=\frac{1}{4}-\frac{1}{2}+1-2+2^2-...=\frac{1/4}{1+2}=\frac{1}{12}$.
**Reason (R) :** Sum of $n$ terms of a G.P. with first term as '$a$' and common ratio as '$r$' is given by $a\left(\frac{r^n-1}{r-1}\right), |r|>1$.

## SECTION - B

**This section comprises of very short answer type questions (VSA) of 2 marks each.**

21. Find the equation of the straight line upon which the length of the perpendicular from the origin is 5 and the slope of this perpendicular is 3/4.

22. In a G.P. of positive terms, if any term is equal to the sum of the next two terms, then find the common ratio of the G.P.

OR

Find the value of $(32)\cdot(32)^{1/6}\cdot(32)^{1/36}\cdot$... to ∞.

23. Find the equation of a line which divides the join of (1, 0) and (3, 0) in the ratio 2 : 1 and perpendicular to it.

24. If the image of the point (3, 8) in the line $px + 3y - 7 = 0$ is the point (− 1, − 4), then find the value of $p$.

OR

Find the angle between the lines
$y=(2-\sqrt{3})(x+5)$ and $y=(2+\sqrt{3})(x-7)$.

25. Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

## SECTION - C

**This section comprises of short answer type questions (SA) of 3 marks each.**

**26.** If $\frac{2}{3}=\left(x-\frac{1}{y}\right)+\left(x^2-\frac{1}{y^2}\right)+\left(x^3-\frac{1}{y^3}\right)+\ldots$ upto $\infty$ and $xy = 2$ and $|x| < 1$, then calculate the values of $x$ and $y$.

**27.** If the two vertices of an equilateral triangle be $(0, 0), (3, \sqrt{3})$, then find the third vertex.

**OR**

A quadrilateral has the vertices at the points $(-4, 2)$, $(2, 6)$, $(8, 5)$ and $(9, -7)$. Show that the mid-points of the sides of this quadrilateral are vertices of a parallelogram.

**28.** Find the sum to infinity of the series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$, where $|x| < 1, |y_1| < 1$.

**29.** Find the equation of the straight line which passes through the point $(1, 2)$ and makes an angle $\theta$ with the positive direction of $X$-axis where $\cos\theta = -\frac{1}{3}$.

**30.** The product of the first three terms of a G.P. is 1000. If we add 6 to its second term and 7 to its third term, the resulting three terms form an A.P. Find the terms of the G.P.

**OR**

Let $a_n$ be the $n^{\text{th}}$ term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$ such that $\alpha \neq \beta$. Prove that the common ratio of the G.P. is $\alpha/\beta$.

**31.** A point moves such that its distance from the point $(4, 0)$ is half that of its distance from the line $x = 16$. Find the locus of that point. Is the point $(2, 4)$ satisfy the obtained locus?

**OR**

Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with positive direction of $X$-axis is 15°. Also, find the $x$ and $y$ intercept of the obtained equation of the line.

## SECTION - D

**This section comprises of long answer type questions (LA) of 5 marks each.**

**32.** In an increasing G.P., the sum of the first and last term is 66, and product of the second and second last term is 128. If the sum of the series is 126, then find the number of terms in the series.

**33.** Two opposite vertices of a square are $(3, 4)$ and $(1, -1)$. Find the coordinates of the other vertices.

**OR**

The mid-points of the sides of a triangle are $(2, 1)$, $(-5, 7)$ and $(-5, -5)$. Find the equations of the sides.

**34.** Find the sum to $n$ terms of the sequence

$$\left(x+\frac{1}{x}\right)^2, \left(x^2+\frac{1}{x^2}\right)^2, \left(x^3+\frac{1}{x^3}\right)^2, \ldots$$

**OR**

Insert four A.M.'s between 1/2 and 3 and prove that $A_1 + A_2 + A_3 + A_4 = 7$. Find the 100th term of the A.P.

**35.** Find the equations of the lines which pass through the point $(3, -2)$ and are inclined at 60° to the line $y = -\sqrt{3}x + 1$.

## SECTION - E

**This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii) and (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.**

**36. Case study 1:** Ram and Ankit are cousins. They study in the same school but in different classes. Ram is older than Ankit. If the ages of Ram and Ankit are $x$ and $y$ respectively.

Based on the above information, answer the following questions.

**(i)** If $n$ geometric means be inserted between $x$ and $y$, then find the $n^{\text{th}}$ geometric mean.

**(ii)** Find the product of $n$ geometric means between $x$ and $1/x$ is.

**(iii)** If arithmetic mean of their ages exceeds the geometric mean of their ages by 2 and Ram is 12 years older than Ankit, then what will be the ages of Ram and Ankit?

**OR**

Find the two geometric means between the number 1 and 64 are

**37.** **Case study 2:** Raju gets an assignment for home work. The assignment has following five problems on progressions. Based on the above information, answer the following questions.

**(i)** If $x, y, z$ are in G.P., then prove that $x(y^2 + z^2) = z(x^2 + y^2)$.

**(ii)** In a G.P., sum of second and fifth term is 216 and the ratio of forth and sixth term 1 : 4, then find its first term (given that all terms are integers).

**(iii)** Four numbers are in arithmetic progression. The sum of first and last term is 8 and the product of middle term is 15, then find the series.

**OR**

If the 10th term of a geometric progression is 9 and 4th term is 4, then find 7th term.

**38.** **Case study 3:** Ravi who was a hotel manager, got a new job, where he had to travel from a boat, in order to reach to the hotel. One day, he was passing through a bridge. Ravi noticed that the bridge built from triangular shaped blocks. Its vertices are $A(-2, 3)$, $B(4, 5)$, $C(2, 1)$ and $D$ is a mid point of $BC$.

Based on the above information, answer the following questions.

**(i)** Find the equation of the line segment $BC$ and $AC$.

**(ii)** What will be the measurement of angle $B$?

## SOLUTIONS

**1.** **(c)** **2.** **(d)** **3.** **(d)**

**4.** **(b) :** Given, $(x_1, y_1) \equiv (3, -1)$ and $(x_2, y_2) \equiv (4, -2)$

Slope of line, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2+1}{4-3} = -1$

$\Rightarrow$ $\tan\theta = -1 \Rightarrow \tan\theta = -\tan 45°$

$\Rightarrow$ $\tan\theta = \tan(180° - 45°) \Rightarrow \tan\theta = \tan 135°$

$\Rightarrow$ $\theta = 135°$

**5.** **(a)** **6.** **(a)**

**7.** **(b) :** We have, an A.P. with $a = 300000$, $d = 10000$ and $n = 20$

$$\therefore \quad S_{20} = \frac{20}{2}[600000 + 19 \times 10000] = ₹\ 7900000$$

**8.** **(a) :** Here, the slope of the line, $m = \tan 30° = \dfrac{1}{\sqrt{3}}$ and the point $(x_1, y_1)$ is (2, 3). Now, by using point slope formula of the equation of a line, we have

$$y - 3 = \frac{1}{\sqrt{3}}(x-2) \text{ or } x - \sqrt{3}y + (3\sqrt{3} - 2) = 0$$

**9.** **(a)**

**10.** **(a) :** The equation of the line $x - y + 3 = 0$ can be written as $y = x + 3 \Rightarrow m = \tan\theta = 1$ and hence $\theta = 45°$.

**11.** **(c) :** Here $a = 5$ and $r = 5$

Thus, $a_{10} = 5(5)^{10-1} = 5\,(5)^9 = 5^{10}$

and $a_n = ar^{n-1} = 5(5)^{n-1} = 5^n$

**12.** **(c)**

**13.** **(a) :** Let 131072 be the $n$th term of the given G.P.

Here $a = 2$ and $r = 4$

$\therefore$ $131072 = a_n = 2(4)^{n-1} \Rightarrow 65536 = 4^{n-1} \Rightarrow 4^8 = 4^{n-1}$

$\Rightarrow$ $n - 1 = 8 \Rightarrow n = 9$

Hence, 131072 is the 9th term of the G.P.

**14.** **(c) :** The distance of the point $P(1, -3)$ from the line $2y - 3x - 4 = 0$ is the length of perpendicular from the point to the line, which is given by

$$\left|\frac{2(-3) - 3 - 4}{\sqrt{13}}\right| = \sqrt{13} \text{ units}$$

**15.** **(d) :** We know that A.M. ≥ G.M. for positive numbers.

Therefore, $\dfrac{3^x + 3^{1-x}}{2} \ge \sqrt{3^x \cdot 3^{1-x}} \Rightarrow 3^x + 3^{1-x} \ge 2\sqrt{3}$

**16.** **(b) :** Let $\theta$ be the angle between the given lines.

$m_1$ = Slope of the line joining (0, 0) and (2, 3) $= \dfrac{3-0}{2-0} = \dfrac{3}{2}$

$m_2$ = Slope of the line joining (2, –2) and (3, 5)

$$= \frac{5+2}{3-2} = 7$$

$$\because \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \frac{11/2}{23/2} = \frac{11}{23} \Rightarrow \theta = \tan^{-1}\frac{11}{23}$$

**17.** **(c)**

**18.** **(a) :** Slope of the line joining (3, 4) and $(-2, a) = -2/5$

$$\Rightarrow \quad \frac{a-4}{-2-3} = \frac{-2}{5} \Rightarrow \frac{a-4}{-5} = \frac{-2}{5} \Rightarrow a - 4 = 2 \Rightarrow a = 6$$

**19.** **(a)** **20.** **(d)**

**21.** Given that $m_1 = 3/4$ (slope of perpendicular)

Slope of required line $m_2 = -4/3$

$\therefore$ Equation of line having slope $m_2$ and $y$-intercept say '$c$' is given by $y = m_2x + c \Rightarrow y = \frac{-4}{3}x + c$

Now, length of perpendicular from the origin = 5

$$\Rightarrow \left|\frac{3c}{\sqrt{3^2+4^2}}\right| = 5 \Rightarrow c = \frac{\pm 25}{3}$$

$\therefore$ Required line is $3y + 4x \pm 25 = 0$

**22.** Let $a$ be the first term and $r$ be the common ratio of the G.P.

According to the question, $a_n = a_{n+1} + a_{n+2}$

$\Rightarrow ar^{n-1} = ar^n + ar^{n+1} \Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{\sqrt{5}-1}{2} \quad [\because r > 0]$$

$$\Rightarrow r = 2\left(\frac{\sqrt{5}-1}{4}\right)$$

**OR**

We have, $(32)\cdot(32)^{1/6}\cdot(32)^{1/36}\cdot$... to $\infty$

$$= (32)^{1+\frac{1}{6}+\frac{1}{36}+\ldots\text{to }\infty} = 32^{\frac{1}{1-1/6}}$$

[$\because$ Power of 32 is in the form of infinite G.P. with first term, $a = 1$ and common ratio, $r = 1/6$]

$$= 32^{6/5} = \left((2)^5\right)^{6/5} = 2^6 = 64$$

**23.** Let $C$ be the point which divides the join of $A(1, 0)$ and $B(3, 0)$ in the ratio 2 : 1. Then, the coordinates of $C$ are

$$\left(\frac{2\times3+1\times1}{2+1}, \frac{2\times0+1\times0}{2+1}\right) = \left(\frac{7}{3}, 0\right)$$

Since $AB$ is along $X$-axis, therefore a line perpendicular to $AB$ is parallel to $Y$-axis. As it passes through $C(7/3, 0)$, therefore its equation is $x = 7/3 \Rightarrow 3x = 7$

Hence, the equation of the required line is $3x = 7$.

**24.** Mid-point of the line joining (3, 8) and (–1, – 4) is $\left(\frac{3-1}{2}, \frac{8-4}{2}\right) = (1, 2)$. Clearly, this point will lie on the line $px + 3y - 7 = 0$.

So, we have $p(1) + 3(2) - 7 = 0 \Rightarrow p = 1$

**OR**

*Refer to answer 11, Page no. 266, MTG 100 percent Class-11.*

**25.** Let $G_1, G_2, G_3$ be three numbers between 1 and 256.

$\therefore$ 1, $G_1$, $G_2$, $G_3$, 256 are in G.P. Here, first term, $a = 1$. Let $r$ be the common ratio.

Now, $a_5 = 256 \Rightarrow ar^4 = 256 \Rightarrow 1\cdot r^4 = 256 \Rightarrow r = 4$

For $r = 4$, $G_1 = ar = 4$, $G_2 = ar^2 = 1\cdot(4)^2 = 16$;

$G_3 = ar^3 = 1\cdot(4)^3 = 64$

Hence, the required numbers are 4, 16, 64.

**26.** We have,

$$\frac{2}{3} = \left(x - \frac{1}{y}\right) + \left(x^2 - \frac{1}{y^2}\right) + \left(x^3 - \frac{1}{y^3}\right) + \ldots \text{ up to } \infty$$

$$= (x + x^2 + x^3 + \ldots \infty) - \left[\frac{1}{y} + \frac{1}{y^2} + \frac{1}{y^3} + \ldots\infty\right]$$

$$= \frac{x}{1-x} - \left(\frac{1/y}{1-1/y}\right) = \frac{x}{1-x} - \frac{1}{y-1} = \frac{x(y-1) - 1(1-x)}{(1-x)(y-1)}$$

$\Rightarrow 2x + 2y - 6 = 3 \Rightarrow 2x + 2y = 9 \Rightarrow x + y = 9/2$ ...(i)

Now, $(x-y)^2 = (x+y)^2 - 4xy = \left(\frac{9}{2}\right)^2 - 4(2) = \frac{49}{4}$

$\Rightarrow x - y = \pm 7/2$ ...(ii)

Solving (i) and (ii), we get $x = 4$, $y = 1/2$ (rejected)

or $x = 1/2$, $y = 4$

**27.** Let $B(x, y)$ be the third vertex of equilateral triangle $OAB$, where $O = (0, 0)$ and $A = (3, \sqrt{3})$.

$\Rightarrow OA = AB = OB \Rightarrow OA^2 = AB^2 = OB^2$

$\Rightarrow OA^2 = OB^2$ and $OB^2 = AB^2$

$\Rightarrow (3-0)^2 + (\sqrt{3}-0)^2 = (x-0)^2 + (y-0)^2$

and $(x-0)^2 + (y-0)^2 = (x-3)^2 + (y-\sqrt{3})^2$

$\Rightarrow x^2 + y^2 = 12$ and $x^2 + y^2 = x^2 + 12 + y^2 - 6x - 2\sqrt{3}y$

$\Rightarrow x^2 + y^2 = 12$ ...(i) and $3x + \sqrt{3}y = 6$ ...(ii)

Put (ii) in (i), we get, $x^2 + \left(\frac{6-3x}{\sqrt{3}}\right)^2 = 12$

$\Rightarrow 3x^2 + (6-3x)^2 = 36 \Rightarrow 12x^2 - 36x = 0 \Rightarrow x = 0, 3$

$\Rightarrow y = 2\sqrt{3}$ and $-\sqrt{3}$. Hence, $B = (0, 2\sqrt{3})$ or $(3, -\sqrt{3})$.

**OR**

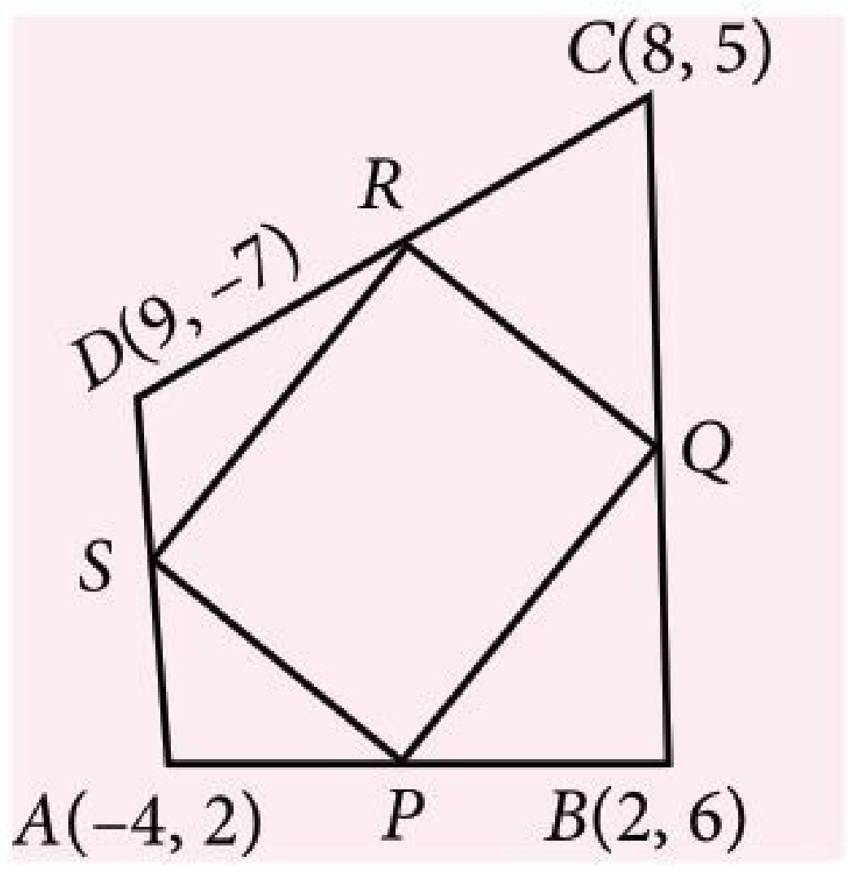
Let $A = (-4, 2)$, $B = (2, 6)$, $C = (8, 5)$ and $D = (9, -7)$. Let $P, Q, R, S$ be the mid-points of the sides $AB$, $BC$, $CD$ and $DA$ respectively. Then

$P = \left(\frac{-4+2}{2}, \frac{2+6}{2}\right)$ *i.e.*, $P = (-1, 4)$

$Q = \left(\frac{2+8}{2}, \frac{6+5}{2}\right)$ *i.e.*, $Q = \left(5, \frac{11}{2}\right)$

$R = \left(\frac{8+9}{2}, \frac{5-7}{2}\right)$ *i.e.*, $R = \left(\frac{17}{2}, -1\right)$

$S = \left(\frac{9-4}{2}, \frac{-7+2}{2}\right)$ *i.e.*, $S = \left(\frac{5}{2}, -\frac{5}{2}\right)$



$m_1$ = slope of $PQ$ = 1/4 ; $m_2$ = slope of $RS$ = 1/4

$m_1$ = slope of $PS$ = –13/7 ; $m_4$ = slope of $QR$ = –13/7

Since $m_1 = m_2 \therefore PQ \parallel SR$

and $m_3 = m_4 \therefore PS \parallel QR$

$\therefore$ $PQRS$ is a parallelogram.

**28.** Given series is : $(x+y)+(x^2+xy+y^2)$
$+(x^3+x^2y+xy^2+y^3)+\ldots$

$$=\frac{1}{(x-y)}[(x-y)(x+y)+(x-y)(x^2+xy+y^2)+(x-y)(x^3+x^2y+xy^2+y^3)+\ldots]$$

$$=\frac{1}{(x-y)}[(x^2+x^3+x^4+\ldots\infty)-(y^2+y^3+y^4+\ldots\infty)]$$

$$=\frac{1}{(x-y)}\left[\frac{x^2}{1-x}-\frac{y^2}{1-y}\right]=\frac{1}{(x-y)}\left[\frac{x^2-x^2y-y^2+xy^2}{(1-x)(1-y)}\right]$$

$$=\frac{1}{(x-y)}\left[\frac{(x^2-y^2)-xy(x-y)}{(1-x)(1-y)}\right]=\frac{x+y-xy}{(1-x)(1-y)}$$

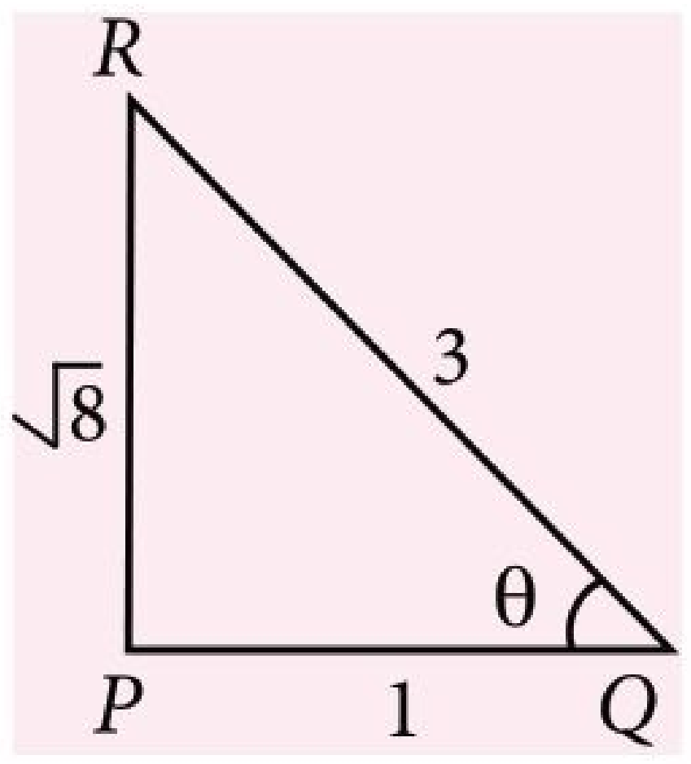
**29.** Here, $\cos\theta=-1/3$ (a negative number)

$\therefore\ \frac{\pi}{2}<\theta<\pi$

$\therefore\ \tan\theta=-\sqrt{8}=$ slope of the line ($m$).

Now, the equation of the straight line passing through the point $(x_1, y_1)$ having slope $m$ is

$$y-y_1=m(x-x_1)$$



Here $x_1=1, y_1=2$ and $m=-\sqrt{8}$

Therefore, equation of the required line is

$y-2=-\sqrt{8}(x-1)\Rightarrow\sqrt{8}x+y-\sqrt{8}-2=0$

or $2\sqrt{2}x+y-2\sqrt{2}-2=0$ or $2\sqrt{2}x+y-2(\sqrt{2}+1)=0$.

**30.** Let first three terms of G.P. be $a/r, a, ar$.

Here, $\frac{a}{r}\cdot a\cdot ar=1000\Rightarrow a^3=1000\Rightarrow a=10$

According to the question, $\frac{a}{r}, a+6, ar+7$ are in A.P.

$\Rightarrow\ \frac{10}{r}, 16, 10r+7$ are in A.P.

$\Rightarrow\ 2(16)=\frac{10}{r}+10r+7\Rightarrow 32r=10+10r^2+7r$

$\Rightarrow\ 10r^2-25r+10=0\Rightarrow 2r^2-5r+2=0$

$\Rightarrow\ (r-2)(2r-1)=0\Rightarrow r=2$ or $1/2$

Thus, terms of the G.P. are 5, 10, 20 or 20, 10, 5.

**OR**

Let $a$ be the first term and $r$ be the common ratio of the G.P.

$$\sum_{n=1}^{100}a_{2n}=\alpha \text{ and } \sum_{n=1}^{100}a_{2n-1}=\beta$$

$\Rightarrow\ a_2+a_4+\ldots+a_{200}=\alpha$ and $a_1+a_3+\ldots+a_{199}=\beta$

$\Rightarrow\ ar+ar^3+\ldots+ar^{199}=\alpha$ and $a+ar^2+\ldots+ar^{198}=\beta$

$\Rightarrow\ ar\left\{\frac{1-(r^2)^{100}}{1-r^2}\right\}=\alpha\ \ldots(i)$ and $a\left\{\frac{1-(r^2)^{100}}{1-r^2}\right\}=\beta\ \ldots(ii)$

Dividing (i) by (ii), we get $r=\alpha/\beta$.

**31.** Let $(h, k)$ be the coordinates of the moving point. Then, we have

$$\sqrt{(h-4)^2+k^2}=\frac{1}{2}\cdot\frac{|h-16|}{\sqrt{1^2+0}}$$

$\Rightarrow\ (h-4)^2+k^2=\frac{1}{4}(h-16)^2$

$\Rightarrow\ 4(h^2-8h+16+k^2)=h^2-32h+256$ or $3h^2+4k^2=192$

Hence, the required locus is given by $3x^2+4y^2=192$

Now, Put $x=2, y=4$ in $3x^2+4y^2$

$\Rightarrow\ 3(2)^2+4(4)^2=12+64=76\neq192$

Hence, the point (2, 4) does not satisfy the equation $3x^2+4y^2=192$.

**OR**

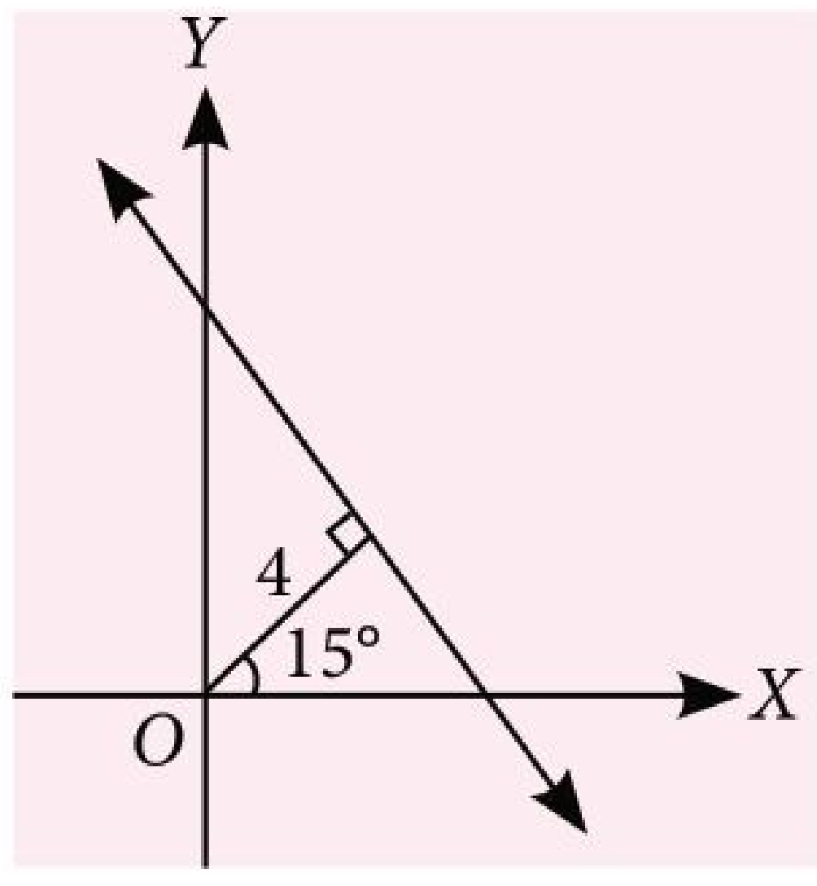
Here, we are given $p=4$ and $\omega=15°$.

Now, $\cos 15°=\frac{\sqrt{3}+1}{2\sqrt{2}}$ and

$\sin 15°=\frac{\sqrt{3}-1}{2\sqrt{2}}$

By the normal form, the equation of the line is

$x\cos 15°+y\sin 15°=4$



$\Rightarrow\ \frac{\sqrt{3}+1}{2\sqrt{2}}x+\frac{\sqrt{3}-1}{2\sqrt{2}}y=4$

$\Rightarrow\ (\sqrt{3}+1)x+(\sqrt{3}-1)y=8\sqrt{2}.$

For $x$-intercept, put $y=0$.

$\therefore\ x=\frac{8\sqrt{2}}{\sqrt{3}+1}=4\sqrt{2}(\sqrt{3}-1)$

For $y$-intercept, put $x=0$.

$\therefore\ y=4\sqrt{2}(\sqrt{3}+1).$

**32.** Let G.P. be $a, ar, ar^2, \ldots, ar^{n-1}$

Given, $a+ar^{n-1}=66\Rightarrow a(1+r^{n-1})=66$ ...(i)

and $ar\cdot ar^{n-2}=a^2\cdot r^{n-1}=128$ ...(ii)

and $\frac{a(r^n-1)}{r-1}=126$ ...(iii)

From (i) and (ii), we have

$a\left(1+\frac{128}{a^2}\right)=66\Rightarrow a^2-66a+128=0\Rightarrow a=64, a=2$

From (ii) when $a=64$, $(64)^2r^{n-1}=128\Rightarrow r^{n-1}=1/32$

[Not valid as G.P. is increasing]

When $a=2$, $(2)^2r^{n-1}=128\Rightarrow r^{n-1}=32$ ...(iv)

Substituting value of $r^{n-1}$ from (iv) in (iii), we get

$\frac{2(32r-1)}{r-1}=126$

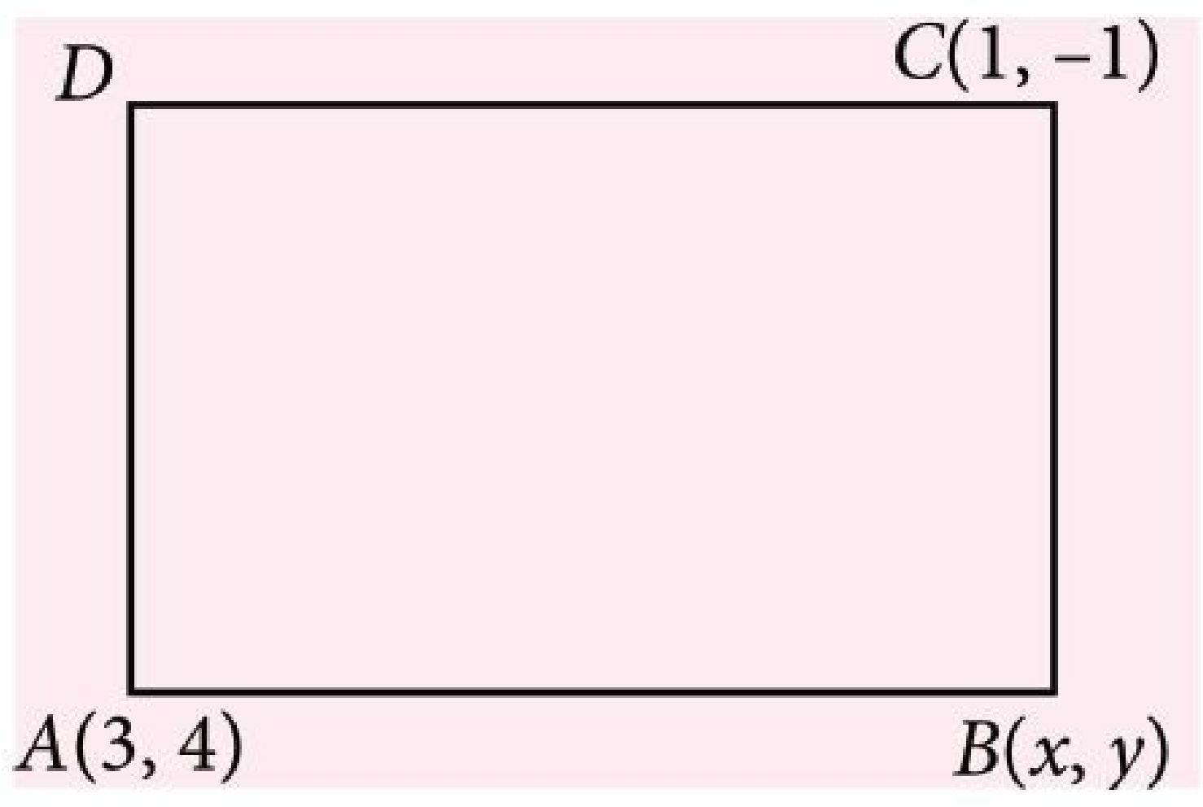
$\Rightarrow \quad 32r - 1 = 63r - 63 \Rightarrow 31r = 62 \Rightarrow r = 2$

Substituting $a = 2$, $r = 2$ in (ii), we get

$(2)^2\,(2)^{n-1} = 128 \Rightarrow 2^{n-1} = 32 = 2^5$

$\Rightarrow \quad n - 1 = 5 \Rightarrow n = 6$

**33.** Let $ABCD$ be a square and let $A(3, 4)$ and $C(1, -1)$ be the given angular points. Let $B(x, y)$ be the unknown vertex.



Then, $AB = BC$

$\Rightarrow \quad AB^2 = BC^2$

$\Rightarrow \quad (x - 3)^2 + (y - 4)^2 = (x - 1)^2 + (y + 1)^2$

$\Rightarrow \quad 4x + 10y - 23 = 0$

$$\Rightarrow \quad x = \frac{23 - 10y}{4} \qquad \ldots\text{(i)}$$

Applying Pythagoras theorem in triangle $ABC$, we obtain $AB^2 + BC^2 = AC^2$

$\Rightarrow \quad (x-3)^2 + (y-4)^2 + (x-1)^2 + (y+1)^2 = (3-1)^2 + (4+1)^2$

$\Rightarrow \quad x^2 + y^2 - 4x - 3y - 1 = 0 \qquad \ldots\text{(ii)}$

Substituting the value of $x$ from (i) into (ii), we get

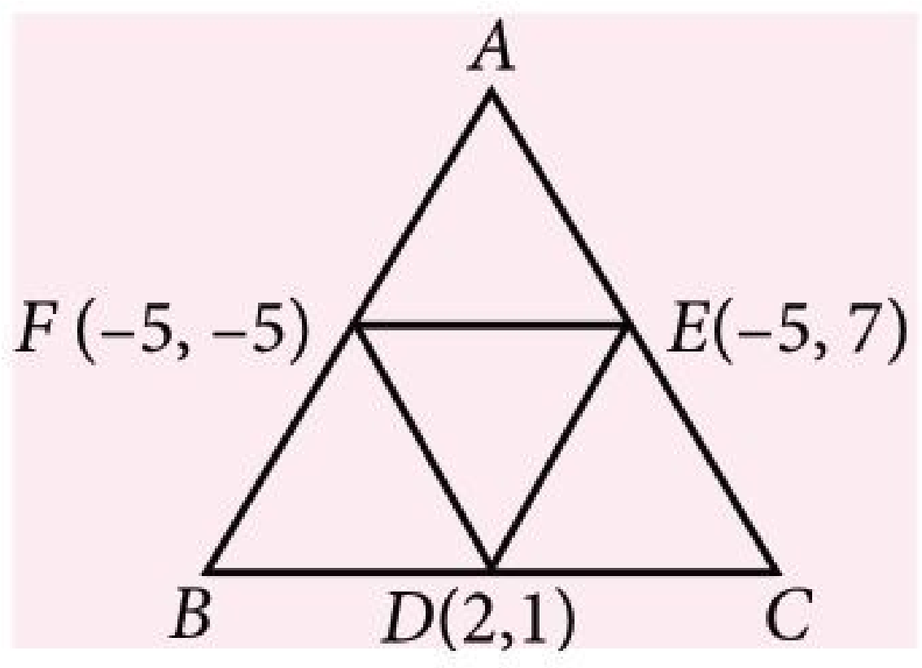
$$y = \frac{1}{2} \text{ or } y = \frac{5}{2}$$

Putting $y = \frac{1}{2}$ and $y = \frac{5}{2}$ in (i), we get $x = \frac{9}{2}$ and $x = -\frac{1}{2}$ respectively.

Hence, the required vertices of the square are (9/2, 1/2) and (−1/2, 5/2).

**OR**

Let $ABC$ be a triangle and $D(2, 1)$, $E(-5, 7)$ and $F(-5, -5)$ be the mid-points of sides $BC$, $CA$ and $AB$ respectively.



Equation of $BC$ :

$$\text{Slope of } FE = \frac{7+5}{-5+5} \text{ (not defined)}$$

$\therefore \quad FE$ is parallel to $Y$-axis.

Since, $EF \parallel BC$

Therefore, $BC$ is also parallel to $y$-axis. $\quad [\because BC \parallel FE]$

Let the equation of $BC$ be $x = k \qquad \ldots\text{(i)}$

Since, line (i) passes through $D(2, 1)$ $\therefore 2 = k$

From (i), equation of line $BC$ will be $x = 2$

Equation of $AB$ :

$$\text{Slope of } DE = \frac{1-7}{2+5} = -\frac{6}{7}$$

Since $AB \parallel DE$ $\therefore$ Slope of $AB = -6/7$

Since $AB$ passes through $F(-5, -5)$ and has slope $-6/7$, therefore its equation will be $y + 5 = -\frac{6}{7}(x + 5)$

$\Rightarrow \quad 7y + 35 = -6x - 30 \Rightarrow 6x + 7y + 65 = 0$

Equation of $AC$ : Slope of $FD = \frac{-5-1}{-5-2} = \frac{6}{7}$

Since $AC \parallel FD$ $\therefore$ Slope of $AC = 6/7$.

Since $AC$ passes through $E(-5, 7)$ and has slope 6/7, therefore its equation will be $y - 7 = \frac{6}{7}(x + 5)$

$\Rightarrow \quad 7y - 49 = 6x + 30 \Rightarrow 6x - 7y + 79 = 0.$

**34.** Let $S_n$ denote the sum to $n$ terms of the given sequence. Then,

$$S_n = \left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \ldots + \left(x^n + \frac{1}{x^n}\right)^2$$

$$= \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^4 + \frac{1}{x^4} + 2\right) + \ldots + \left(x^{2n} + \frac{1}{x^{2n}} + 2\right)$$

$$= (x^2 + x^4 + x^6 + \ldots + x^{2n}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \ldots + \frac{1}{x^{2n}}\right)$$

$$+ (2 + 2 + 2 + \ldots + n \text{ terms})$$

$$= x^2\left(\frac{(x^2)^n - 1}{x^2 - 1}\right) + \frac{1}{x^2}\left(\frac{(1/x^2)^n - 1}{(1/x^2) - 1}\right) + 2n$$

$$= x^2\left(\frac{x^{2n} - 1}{x^2 - 1}\right) + \frac{1}{x^{2n}}\left(\frac{1 - x^{2n}}{1 - x^2}\right) + 2n$$

$$= x^2\left(\frac{x^{2n} - 1}{x^2 - 1}\right) + \frac{1}{x^{2n}}\left(\frac{x^{2n} - 1}{x^2 - 1}\right) + 2n$$

$$\Rightarrow \quad S_n = \left(\frac{x^{2n} - 1}{x^2 - 1}\right)\left(x^2 + \frac{1}{x^{2n}}\right) + 2n$$

**OR**

Let $A_1$, $A_2$, $A_3$, $A_4$ be four A.M.'s between 1/2 and 3. Therefore, 1/2, $A_1$, $A_2$, $A_3$, $A_4$, 3 are in A.P.

$$t_6 = 3 \Rightarrow \frac{1}{2} + (6-1)d = 3 \Rightarrow 3 - \frac{1}{2} = 5d$$

or, $\quad 5d = \frac{5}{2}$ or, $d = \frac{1}{2} \quad \therefore \quad A_1 = a + d = \frac{1}{2} + \frac{1}{2} = 1$

$$A_2 = a + 2d = \frac{1}{2} + 2 \times \frac{1}{2} = \frac{1}{2} + 1 = 1\frac{1}{2}$$

$$A_3 = a + 3d = \frac{1}{2} + 3 \times \frac{1}{2} = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

$$A_4 = a + 4d = \frac{1}{2} + 4 \times \frac{1}{2} = \frac{1}{2} + 2 = 2\frac{1}{2}$$

Also, $A_1 + A_2 + A_3 + A_4 = 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} = 7$

Now, $t_{100} = \frac{1}{2} + (100 - 1) = \frac{1}{2} = 50$

**35.** *Refer to answer 21, Page no. 268, MTG 100 percent Class-11.*

**36.** **(i)** If $n$ geometric means $g_1, g_2, \ldots, g_n$ are to be inserted between two positive real numbers $x$ and $y$, then $x, g_1, g_2 \ldots g_n, y$ are in G.P.

Then, $g_1 = xr, g_2 = xr^2 \ldots\ldots g_n = xr^n$

So, $y = xr^{n+1} \Rightarrow \quad r = \left(\frac{y}{x}\right)^{\frac{1}{(n+1)}}$

Now, $n^{\text{th}}$ geometric mean $g_n = x\left(\frac{y}{x}\right)^{\frac{n}{n+1}}$

**(ii)** We have $x, g_1, g_2, g_3 \ldots\ldots g_n, 1/x$

$$g_1 \cdot g_2 \cdot g_3 \cdot \ldots\ldots g_n = \left(x \times \frac{1}{x}\right)^{n/2} = 1$$

**(iii)** Given that Ram is 12 years older than Ankit and arithmetic mean of their ages exceed the geometric mean of their ages by 2

$\therefore \quad x - y = 12 \qquad \ldots(1)$

and A.M. – G.M = 2 $\qquad \ldots(2)$

$\Rightarrow \quad \frac{x+y}{2} - \sqrt{xy} = 2 \Rightarrow x + y - 2\sqrt{xy} = 4$

$\Rightarrow \quad (\sqrt{x} - \sqrt{y})^2 = 4 \Rightarrow \sqrt{x} - \sqrt{y} = 2 \qquad \ldots(3)$

Now, $x - y = 12$

$(\sqrt{x} + \sqrt{y})\ (\sqrt{x} - \sqrt{y}) = 12$

$\Rightarrow \quad (\sqrt{x} + \sqrt{y}) \times (2) = 12 \Rightarrow \quad \sqrt{x} + \sqrt{y} = 6 \qquad \ldots(4)$

By (3) and (4), $x = 16$ and $y = 4$.

**OR**

Let $y$ and $x$ be two geometric mean such that $1, y, x, 64$ , then

$y^2 = x \quad \ldots(1) \quad$ and $\quad x^2 = 64y \qquad \ldots(2)$

Put $x = y^2$ in(2), we get $y^4 = 64y$

or $\quad y^3 = 64 \Rightarrow y = 4$

Now, by (1), $x = (4)^2 \Rightarrow \quad x = 16$

$\therefore$ Geometric means are 4, 16.

**37. (i)** If $x, y, z$ are in G.P, then $y^2 = x \cdot z$

$\Rightarrow \quad y^2(x - z) = x \cdot z\,(x - z) \Rightarrow y^2x - y^2z = x^2z - xz^2$

$\Rightarrow \quad y^2x + xz^2 = x^2z + y^2z \Rightarrow x(y^2 + z^2) = z(x^2 + y^2)$

**(ii)** If $a$ is the first term and $r$ is the common ratio of a G.P, then

$a, ar, ar^2, ar^3, ar^4 \ldots\ldots$ are the term of G.P.

$t_2 + t_5 = ar + ar^4 = 216$

and $\frac{t_4}{t_6} = \frac{ar^3}{ar^5} = \frac{1}{4} \quad \Rightarrow \quad r = \pm 2$

for $r = 2$, $a(2 + 2^4) = 216 \Rightarrow a = \frac{216}{18} \Rightarrow a = 12$

For $r = -2$, $a(-2 + 2^4) = 216 \Rightarrow \quad a = \frac{216}{14} = \frac{108}{7}$

$\therefore \quad a = 12$ (Since all terms are integers)

**(iii)** Let $A_1\ A_2\ A_3$ and $A_4$ are four numbers in A.P.

Given, $A_1 + A_4 = 8 \qquad \ldots(1)$

$A_2 \cdot A_3 = 15 \qquad \ldots(2)$

We know that the sum of terms equidistant from the beginning and end is constant and is equal to sum of first and last terms.

$\Rightarrow \quad A_2 + A_3 = A_1 + A_4 = 8 \qquad \ldots(3)$

From (2) and (3), $A_2 + \frac{15}{A_2} = 8 \Rightarrow A_2^2 - 8A_2 + 15 = 0$

$\Rightarrow \quad A_2 = 3$ or 5 and $A_3 = 5$ or 3

As, we know $A_2 = \frac{A_1 + A_3}{2} \Rightarrow A_1 = 2A_2 - A_3$

$\Rightarrow \quad A_1 = 2 \times 3 - 5 = 1$ and $A_4 = 8 - A_1 = 7$

Hence, the series is 1, 3, 5, 7 or 7, 5, 3, 1.

**OR**

Accordingly, $ar^9 = 9$ and $ar^3 = 4$

$\Rightarrow \quad r^3 = \frac{3}{2}$ and $a = \frac{8}{3}$

$\therefore \quad 7^{\text{th}}$ term *i.e.*, $ar^6 = \left(\frac{8}{3}\right)\left(\frac{3}{2}\right)^2 = 6.$

**38. (i)** Equation of $BC$ is $y - 1 = \frac{5-1}{4-2}(x-2)$

$\Rightarrow \quad y - 1 = 2x - 4 \Rightarrow 2x - y - 3 = 0$

Equation of $AC$ is

$y - 1 = \frac{3-1}{-2-2}(x-2) \quad \Rightarrow \quad y - 1 = \frac{-1}{2}(x-2)$

$\therefore \quad x + 2y - 4 = 0.$

**(ii)** Equation of $AB$ is $y - 5 = \frac{3-5}{-2-4}(x-4)$

$\Rightarrow 3(y - 5) = x - 4 \Rightarrow 3y - 15 = x - 4 \Rightarrow x - 3y + 11 = 0$

Let $m_1$ = Slope of $AB = \frac{3-5}{-2-4} = \frac{1}{3}$

and $m_2$ = Slope of $BC = \frac{5-1}{4-2} = 2$

Now, $\tan B = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{\frac{1}{3} - 2}{1 + \frac{2}{3}}\right| = 1 \Rightarrow B = 45°$

◆◆

## UNSCRAMBLED WORDS

### OCTOBER 2022

1-d- COMBINATION

2-e- RELATION

3-a- PERMUTATION

4-b- FUNCTION

5-f- MATRIX

6-i- IMAGINARY

7-j- INEQUALITY

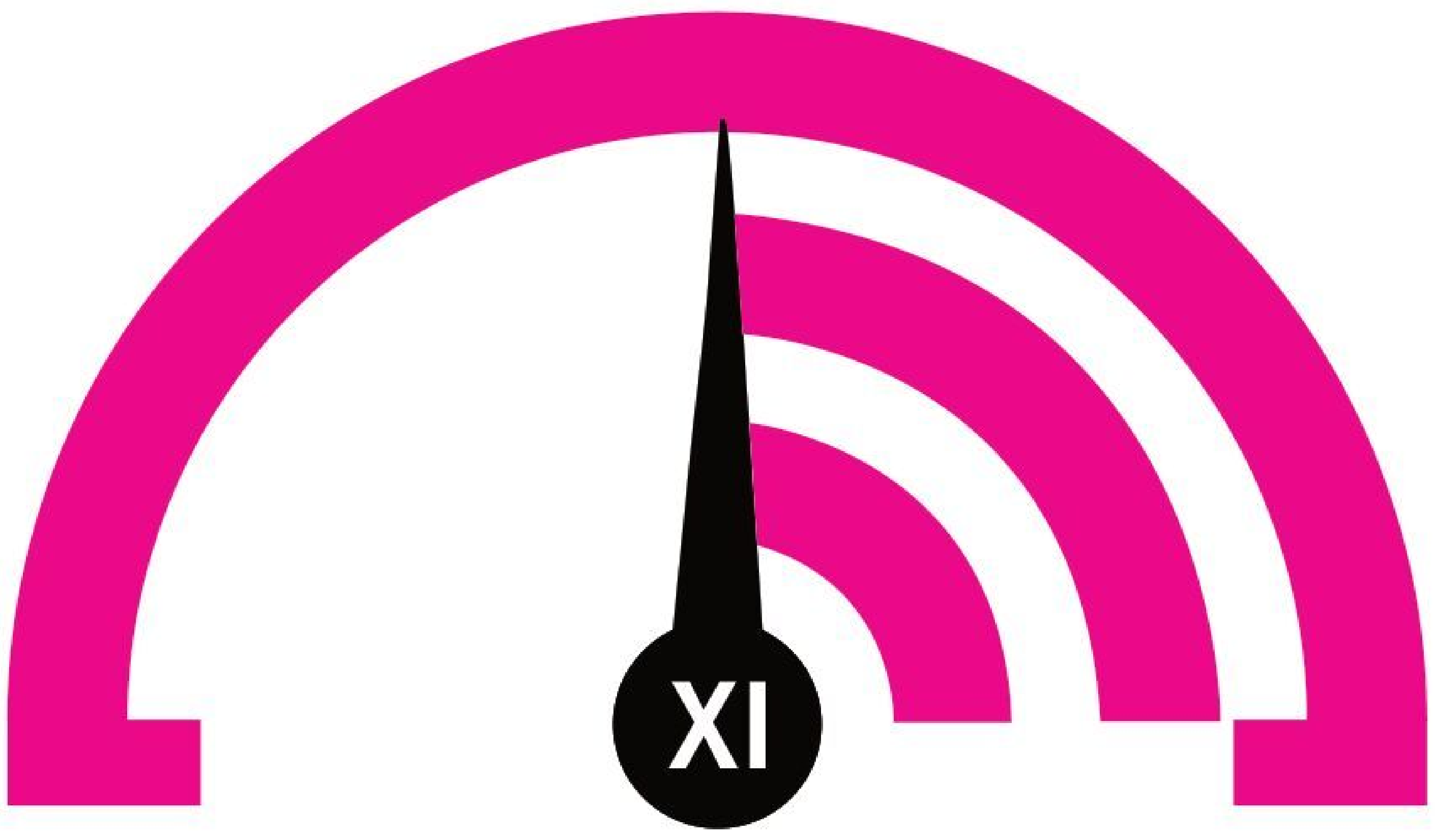
8-c- TRIGONOMETRY

9-g- PROBABILITY

10-h- GEOMETRY

***Winner :** 1. Bubun Sharan, Jamshedpur ; 2. Ankita, Panipat*

# MONTHLY TEST
# DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

**Total Marks : 80**

## Series 4 : Permutations and Combinations | Mathematical Reasoning

**Time Taken : 60 Min.**

### Only One Option Correct Type

**1.** In a 12 storey building 3 persons enter a lift cabin. It is known that they will leave the lift at different storeys. In how many ways can they do so if the lift does not stop at the second storey?
(a) 720 (b) 240 (c) 120 (d) 36

**2.** The negation of the statement "72 is divisible by 2 and 3" is
(a) 72 is not divisible by 2 or 72 is not divisible by 3.
(b) 72 is not divisible by 2 and 72 is not divisible by 3.
(c) 72 is divisible by 2 and 72 is not divisible by 3.
(d) 72 is not divisible by 2 and 72 is divisible by 3.

**3.** The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word COCHIN is
(a) 360 (b) 192 (c) 96 (d) 48

**4.** $(p \to q) \to [(r \vee p) \to (r \vee q)]$ is
(a) a tautology (b) a contradiction
(c) neither a tautology nor a contradiction
(d) None of these

**5.** There are seven greeting cards, each of a different colour and seven envelopes of the same seven colours. The number of ways in which the cards can be put in the envelopes so that exactly four of the cards go into the envelopes of the right colours is
(a) 70 (b) 64 (c) 60 (d) 45

**6.** Which of the following is false?
(a) $p \vee \sim p$ is a tautology
(b) $\sim(\sim p) \leftrightarrow p$ is a tautology
(c) $(p \wedge (p \to q)) \to p$ is a contradiction
(d) $p \wedge \sim p$ is a contradiction

### One or More Than One Option(s) Correct Type

**7.** The number of ways of arranging seven persons (having $A$, $B$, $C$ and $D$ among them) in a row, so that $A$, $B$, $C$ and $D$ are always in order $A$-$B$-$C$-$D$ (not necessarily together) is
(a) 210 (b) 5040 (c) $6 \times {}^7C_3$ (d) ${}^7P_3$

**8.** Consider the proposition : "If the pressure increases, the volume decreases". The equivalent statement of this proposition is
(a) If the pressure does not increase the volume does not decrease
(b) If the volume increases, the pressure decreases
(c) If the volume does not decreases, the pressure does not increase
(d) If the volume decreases, then the pressure increases.

**9.** Number of permutations of the word "AUROBIND" in which vowels appear in an alphabetical order is
(a) ${}^8P_4$ (b) ${}^8C_4$ (c) $4!\ {}^8C_4$ (d) ${}^8C_5\ 5!$

**10.** The contrapositive of the following statement, "If the side of a square doubles, then its area increases four times", is
(a) If the area of a square increases four times, then its side is not doubled.
(b) If the area of a square increases four times, then its side is doubled.
(c) If the area of a square does not increase four times, then its side is not doubled.
(d) If the side of a square is not doubled, then its area does not increase four times.

**11.** Number of ways in which letters of the word ENGINEER can be arranged :

(a) so that the word starts with E but does not end with E is 1600.

(b) so that the word starts with E but does not end with E is 900.

(c) so that vowels occur in alphabetical order is 1050.

(d) so that vowels occur in alphabetical order is 840.

**12.** Check the validity of statements

Statement-1 : $(p \wedge \sim q) \rightarrow (\sim p \wedge q)$

Statement-2 : $(p \rightarrow q) \Leftrightarrow (\sim q \rightarrow \sim p)$

(a) Only Statement-1 is true

(b) Only Statement-2 is true

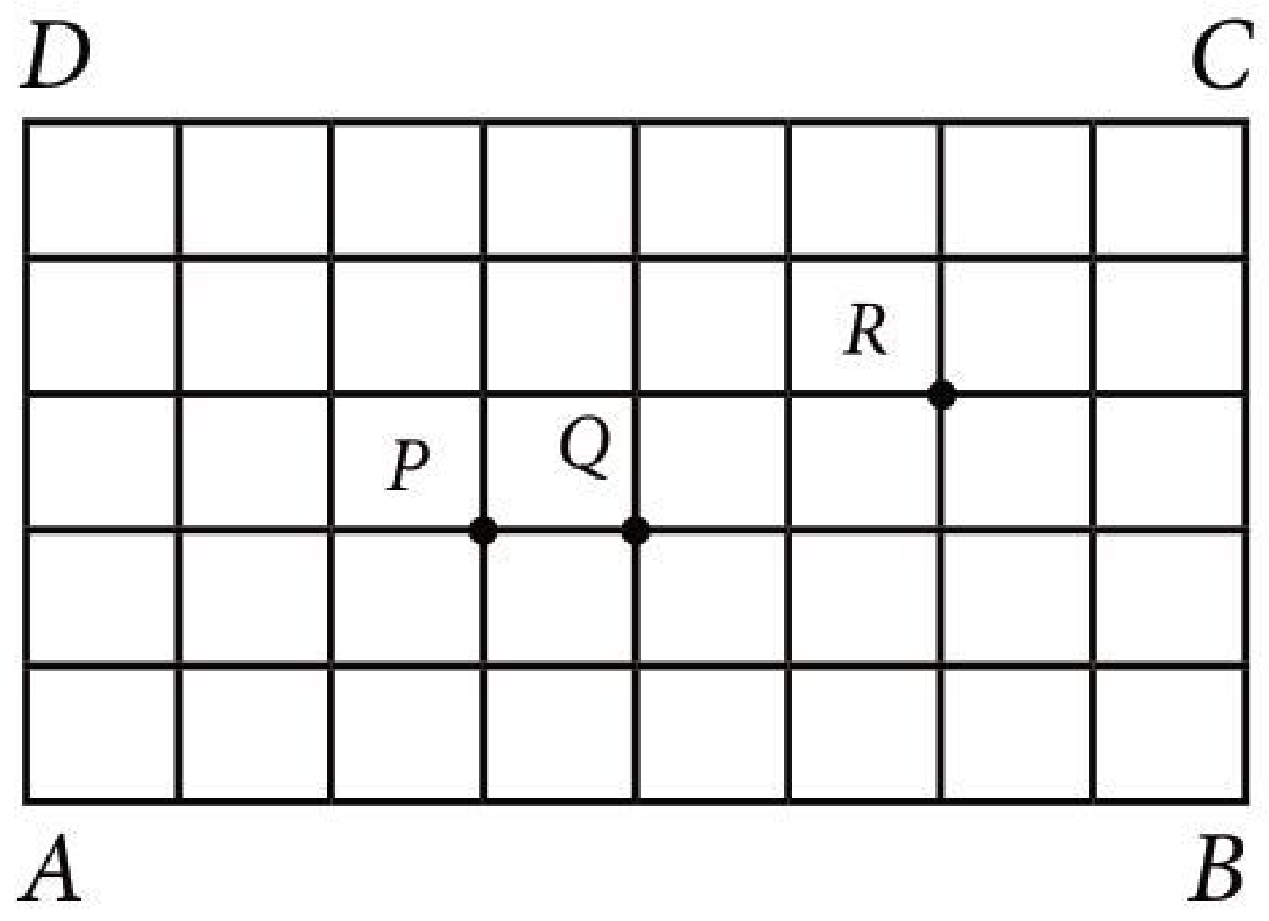
(c) Both Statements are true

(d) Both Statements are false

## Comprehension Type

### Paragraph for Q. No. 13 to 15

Consider the network of equally spaced parallel lines (6 horizontal and 9 vertical) as shown in the following figure.



All small squares are of the same size. A shortest route from $A$ to $C$ is defined as a route consisting 8 horizontal steps ($H$) and 5 vertical steps ($V$). Since any shortest route is a typical arrangement of $8H$ and $5V$, the number of shortest routes is $\frac{13!}{5!8!}$.

**13.** The number of shortest routes through the junction $P$ is

(a) 240 (b) 216 (c) 560 (d) 512

**14.** The number of shortest routes which go following street $PQ$ must be

(a) 324 (b) 350 (c) 512 (d) 256

**15.** The number of shortest routes which pass through junctions $P$ and $R$ is

(a) 144 (b) 240 (c) 216 (d) 256

## Matrix Match Type

**16.** Match the following :

| Column-I | | Column-II | |
|---|---|---|---|
| (P) | Negation of $p$ : '12 is divisible by 3 or 5' is | (1) | a false statement |
| (Q) | Contrapositive of $p \rightarrow q$ is | (2) | a true statement |
| (R) | The statement 'if $x$ is any real, then $x \leq x$' is | (3) | 12 is divisible neither by 3 nor by 5 |
| (S) | The statement '1 is a prime number' is | (4) | $\sim q \rightarrow \sim p$ |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 4 | 3 | 2 | 1 |
| (b) | 3 | 4 | 2 | 1 |
| (c) | 2 | 1 | 4 | 3 |
| (d) | 1 | 4 | 3 | 2 |

## Numerical Answer Type

**17.** If the number of ways in which 6 boys and 5 girls can be arranged on a circular table such that no two girls sit together and two particular boys are always together is $10n^2$, then sum of divisors of $n$ is ______.

**18.** The number of ways of arranging 11 objects $A, B, C, D, E, F, \alpha, \alpha, \alpha, \beta, \beta$ so that every $\beta$ lie between two $\alpha$ (not necessarily adjacent) is $K \times 6! \times {}^{11}C_5$, then $K$ is ______.

**19.** A flight of stairs has 10 steps. A person can go up to steps one at a time, two at a time or any combination of 1's and 2's. The total number of ways in which the person can go up the stairs is ______.

**20.** Eight identical coins are arranged in a row, the number of ways in which the number of tails are equal to the number of heads, is ______.

◆ ◆

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

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No. of questions correct ......

Marks scored in percentage ......

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| 90-75% | GOOD WORK ! | You can score good in the final exam. |
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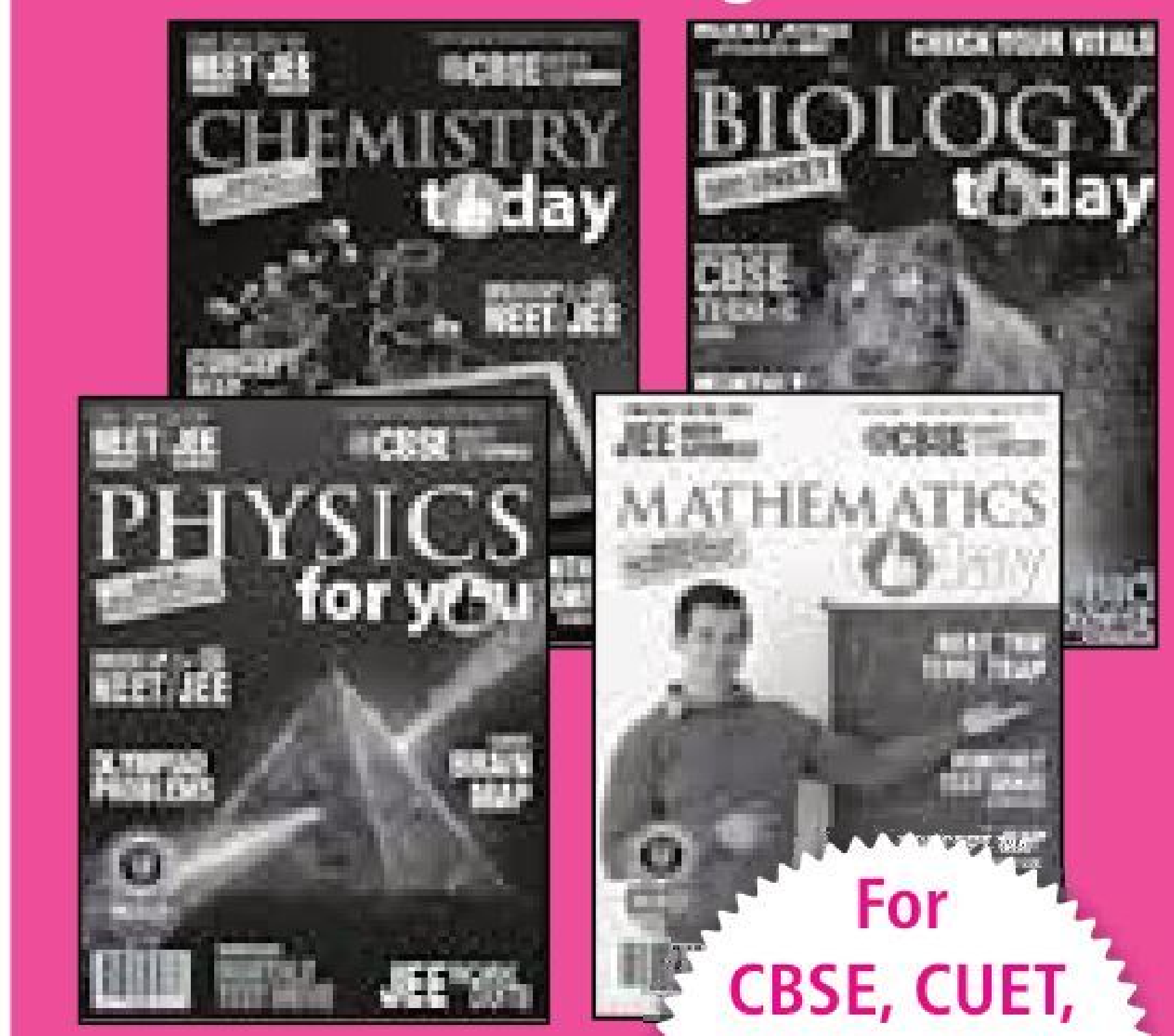

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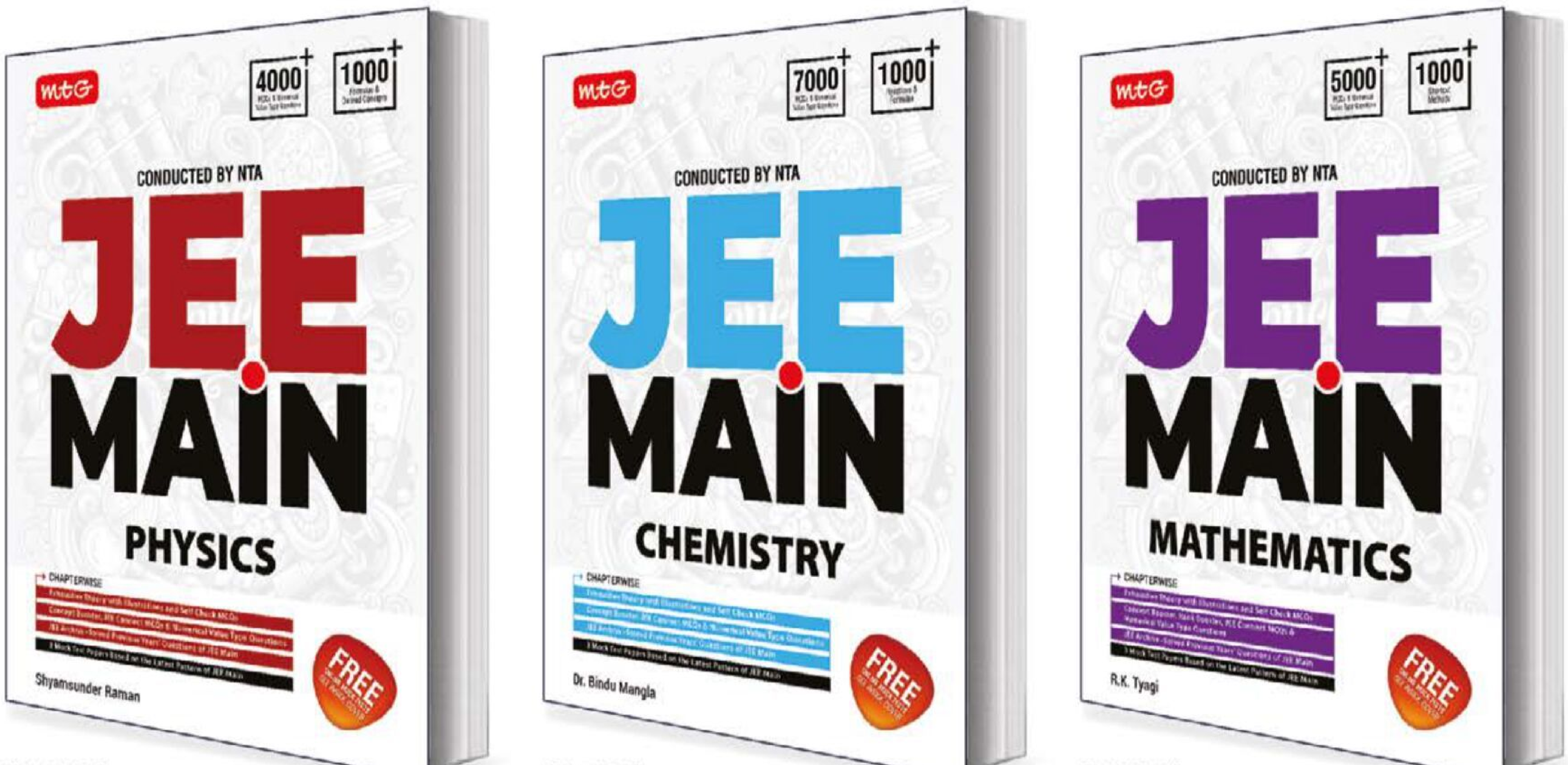

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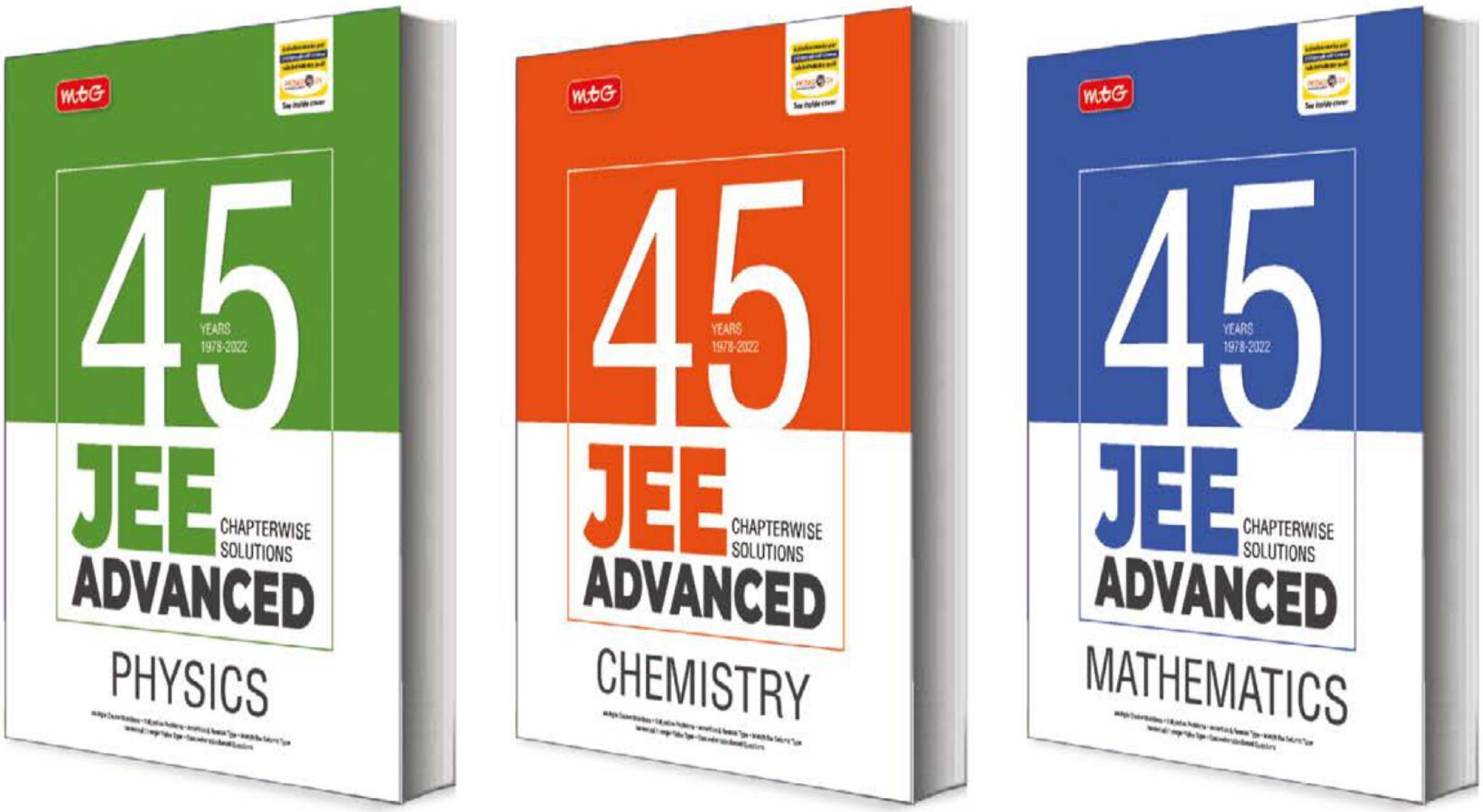

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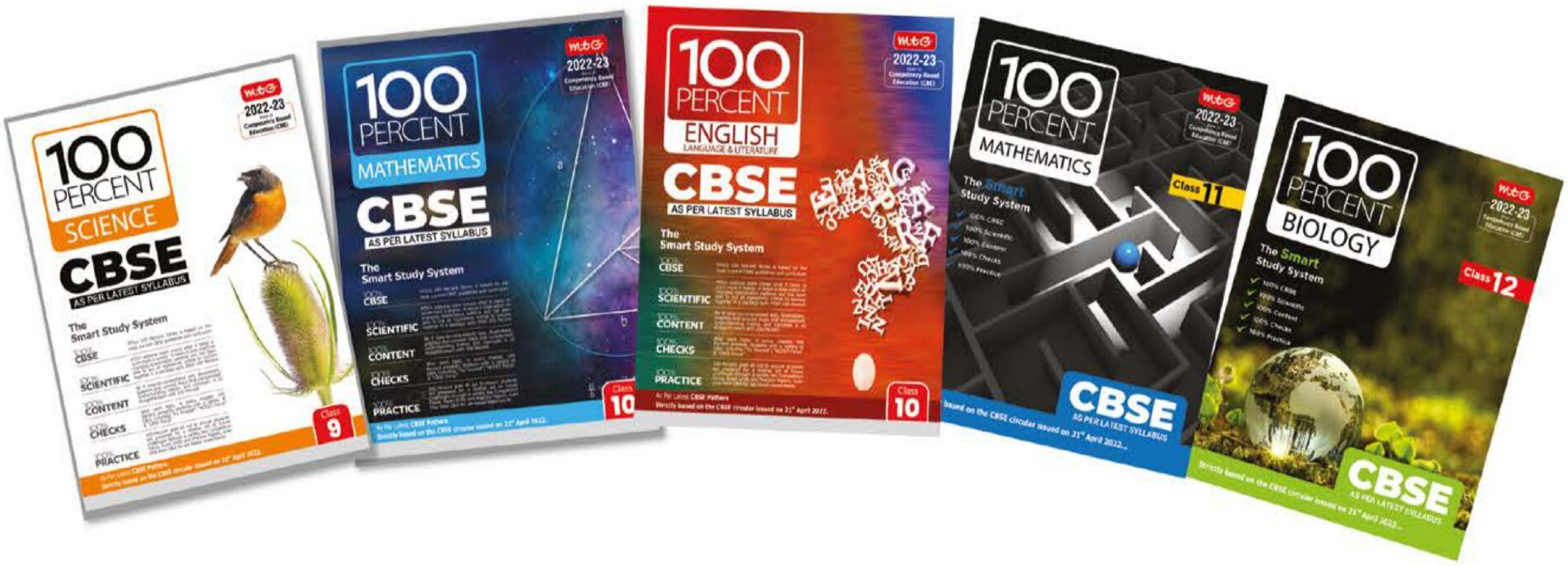

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